

Downhill Simplex Algorithm

This algorithm addresses problems such as $\min(F(\mathbf{x}))$, where F is the objective function and \mathbf{x} is a vector of design variables. The key advantage of downhill simplex is that derivatives of F are NOT needed. Naturally, this leads to slower convergence than gradient-based algorithms. Nelder-Mead is another name of this algorithm, because of the paper “A simplex method for function minimization” published by paper Nelder and Mead in Volume 7 of the Computer Journal in 1965. Their work built on work by Spendley, Hext, and Himsworth published in Volume 4 of Technometrics in 1962 entitled “Sequential Application of Simplex Designs in Optimisation and Evolutionary Operation.”

Operations on a Simplex

A simplex is easy to explain for the case of two design variables; then the simplex is a triangle like the one shown in Figure 1. For each design variable that is added, one point is added to form a simplex. In the downhill simplex algorithm we repeatedly discard one point of the simplex, and add another point to form a new simplex. Ideally, the simplex then steadily moves downhill, closer to the optimum at the bottom of the convex function $F(\mathbf{x})$. There are four ways to reshape the simplex: reflection, expansion, contraction, and shrink. Those may soon be explained here, but for now the algorithm posted at Wikipedia is a nice review: https://en.wikipedia.org/wiki/Nelder-Mead_method.

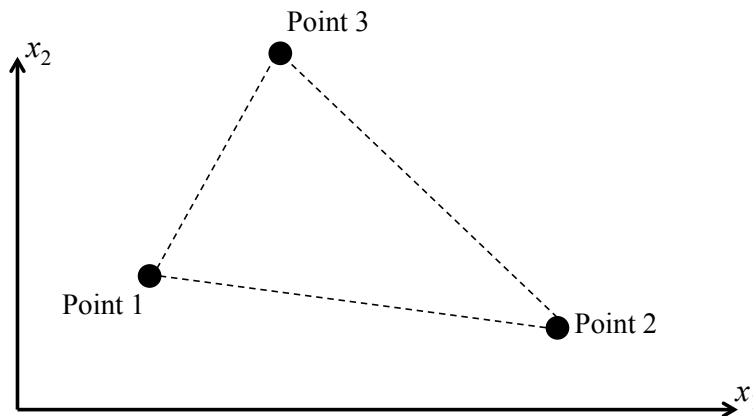


Figure 1: A simplex for the case of two design variables.