Discount Rates

Discounting is an important concept when making decisions that involve comparisons or summations of present and future costs. For several reasons humans prefer costs to materialize in the future instead of now. Equivalently, we want benefits now instead of later. To understand discounting it is useful to know that it is the inverse of compounding. In compounding we calculate the future value of present money; in discounting we calculate the present value of future money. Regardless of application, the factor between present and future money can be very large. Albert Einstein suggested humorously suggested that compounding is the eight wonder of the world and Warren Buffet joked that Queen Isabel should have rather placed in the bank at 4% interest the $30,000 she gave to Columbus to discover America because it would be worth $26 trillion today. This is verified by studying the discount/compound factor, here denoted δ, which says

$$\text{Future compunded value} = \delta \cdot \text{Present value} \tag{1}$$

and conversely

$$\text{Present discounted value} = \frac{\text{Future value}}{\delta} \tag{2}$$

Discrete vs. Continuous

The expression for the factor δ depends on whether we use discrete or continuous compounding/discounting. In practical terms, discrete compounding means that we get the accumulated interest at regular intervals, e.g., once a year so that interest on that extra money only starts accumulating after that. Then the formula is

$$\begin{aligned} \delta &= (1+r)\cdot(1+r)\cdots(1+r) \\ &= (1+r)^n \end{aligned} \tag{3}$$

where r=interest rate per time interval, e.g., annual interest rate, and n=number of intervals, e.g., number of years. The continuous version is

$$\delta = e^{r \cdot t} \tag{4}$$

where t=time period measured in the same unit of time that the interest rate is specified in. The difference between the discrete and continuous version is small but continuous compounding is better: with 3% interest for 50 years you have $\delta = e^{(0.03)(50)} = 4.48$ times the original money and "only" $\delta = (1+0.03)^{50} = 4.38$ with discrete compounding.

Selecting Lifecycle Duration

Table 1 shows the value of δ for combinations of rate and period using the continuous approach. From that table one can state facts like an annual price growth of 7% for real estate leading to a doubling of prices in 10 years. Similarly, using a discount rate near 5%

implies that a cost occurring in 50 years does not even mean a tenth of what the same cost would today. Such considerations can enter into the selection of lifecycle duration for the analysis of a building. Suppose we are confident the building will be around for 50 years, and that we wish to apply a 5% discount rate to all costs. Costs accruing beyond the 50-year time horizon will enter with less than a twelfth of its future value in our calculations. That means it matters little if we select 50 years or 60 years as lifecycle duration.

Table 1: Compound/discount factors.

	1%	2%	3%	4%	5%	6%	7%	8%	9%	10%	15%	20%	25%
1 year	1.0	1.0	1.0	1.0	1.1	1.1	1.1	1.1	1.1	1.1	1.2	1.2	1.3
2 years	1.0	1.0	1.1	1.1	1.1	1.1	1.2	1.2	1.2	1.2	1.3	1.5	1.6
3 years	1.0	1.1	1.1	1.1	1.2	1.2	1.2	1.3	1.3	1.3	1.6	1.8	2.1
4 years	1.0	1.1	1.1	1.2	1.2	1.3	1.3	1.4	1.4	1.5	1.8	2.2	2.7
5 years	1.1	1.1	1.2	1.2	1.3	1.3	1.4	1.5	1.6	1.6	2.1	2.7	3.5
6 years	1.1	1.1	1.2	1.3	1.3	1.4	1.5	1.6	1.7	1.8	2.5	3.3	4.5
7 years	1.1	1.2	1.2	1.3	1.4	1.5	1.6	1.8	1.9	2.0	2.9	4.1	5.8
8 years	1.1	1.2	1.3	1.4	1.5	1.6	1.8	1.9	2.1	2.2	3.3	5.0	7.4
9 years	1.1	1.2	1.3	1.4	1.6	1.7	1.9	2.1	2.2	2.5	3.9	6.0	9.5
10 years	1.1	1.2	1.3	1.5	1.6	1.8	2.0	2.2	2.5	2.7	4.5	7.4	12.2
15 years	1.2	1.3	1.6	1.8	2.1	2.5	2.9	3.3	3.9	4.5	9.5	20.1	42.5
20 years	1.2	1.5	1.8	2.2	2.7	3.3	4.1	5.0	6.0	7.4	20.1	54.6	148.4
25 years	1.3	1.6	2.1	2.7	3.5	4.5	5.8	7.4	9.5	12.2	42.5	148.4	518.0
50 years	1.6	2.7	4.5	7.4	12.2	20.1	33.1	54.6	90.0	148.4	1,808	22,026	268,337
75 years	2.1	4.5	9.5	20.1	42.5	90.0	190.6	403.4	854.1	1,808	76,880	3,269,017	139,002,156
100 years	2.7	7.4	20.1	54.6	148.4	403.4	1,097	2,981	8,103	22,026	3,269,017	485,165,195	72,004,899,337

Sudden or Continuously Accumulating Future Costs

The formulas presented above apply to costs or benefits at a particular time instant in the future. Now consider a cost, c_{rate}, which accumulates continuously as a cost per unit time. The present value of that cost is

$$c_{\text{present}} = \int_{t_1}^{t_2} c_{\text{rate}}(t) \cdot e^{-r \cdot t} \, dt \tag{5}$$

where t_1 and t_2 are the future start and stop times of the cost. If the cost-rate is constant then c_{rate} can be pulled out of the integral so the factor δ can be quantified:

$$\delta = \int_{t_1}^{t_2} e^{r \cdot t} \, dt = \frac{e^{r \cdot t_2} - e^{r \cdot t_1}}{r} \tag{6}$$

Rate Types

A challenging aspect of discounting is the determination of the interest rate, r. Four rates are relevant in this discussion:

r_n = nominal interest rate for money saved or invested
r_i = rate of inflation
r_r = real interest rate
r_p = pure time preference rate

What matters when invested money is compounded with r_n is what is left after inflation has taken some of that profit. That is the reason Irving Fisher postulated that we use

$$r_r = r_n - r_i \tag{7}$$

For the same reason it is appropriate to use r_r in discounting, because inflation will eat away at money invested to pay for future costs.

Canadian Rates

The variation in some Canadian rates is shown in Figure 1. The prime rate minus 2% is selected as the nominal rate because of data availability, and because the nominal bank rate is usually a couple of percentage points below prime. The black line in Figure 1 shows the variation in the real interest rate. The average real interest rate, calculated with the shown values, is precisely 1%. The average in our millennium is near zero, and recently it has been negative.

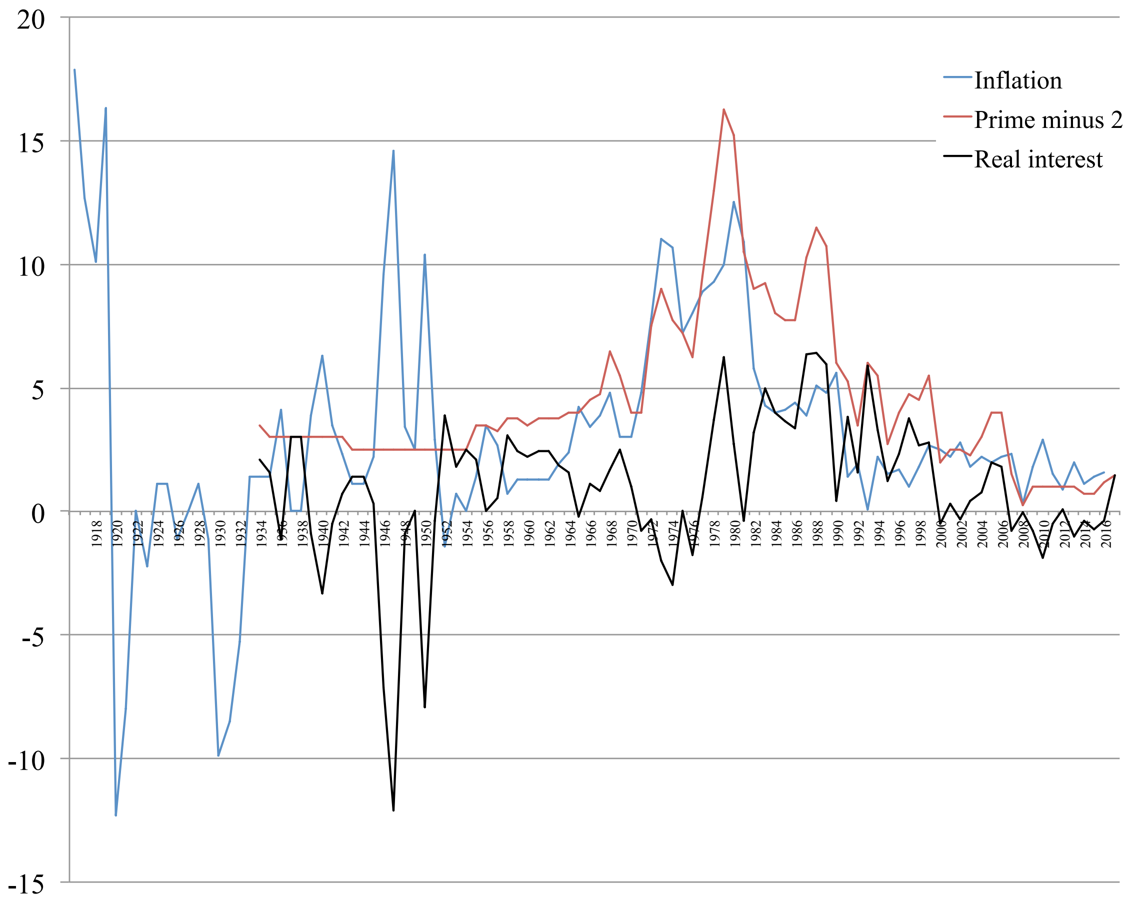

Figure 1: Canadian rates.

Assessing Future Costs Today

When we today estimate the future cost of, say, earthquake damage we use present-day construction cost tables. By the time the cost occurs it will have increased by inflation. Furthermore, as postulated by Fisher in Eq. (7), inflation will eat away at the profit of any money put aside for paying for the damage. This means the discount factor on the cost estimated today is

$$\delta = e^{(r_n - 2r_i)\cdot t} \tag{8}$$

Discounting Environmental Costs

When we expect to receive an invoice for a future cost, then it is financially rational to discount it before it is compared or added to present costs. However, some costs are not paid by invoice. Examples are environmental impacts and human injuries; should those costs be discounted, i.e., reduced, before they are entered into the total lifecycle cost we calculate today? If yes, which rate should be used? The environmental field is one in which that discussion is held:

- Finnveden (1997), Valuation Methods Within LCA – Where are the Values?
- Hellweg, Hofstetter, Hungerbühler (2003), Should Current Impacts be Weighted Differently than Impacts Harming Future Generations?
    - Discounting because of pure time preference is not acceptable; discounting because of capital productivity might be accepted:
    - ... if it is possible to express the damage in monetary units
    - ... if it is possible to compensate future generations for the damage
    - ... if they would be satisfied with such compensation
- Nishijima, Straub, Faber (2007), Inter-generational distribution of the life-cycle cost of an engineering facility
- Stern (2007), The Economics of Climate Change: The Stern Review
- Nordhaus (2007), A Review of the Stern Review on the Economics of Climate Change
- Kula & Evans (2010), Dual discounting in cost-benefit analysis for environmental impacts
- Moxnes (2014), Discounting, climate and sustainability