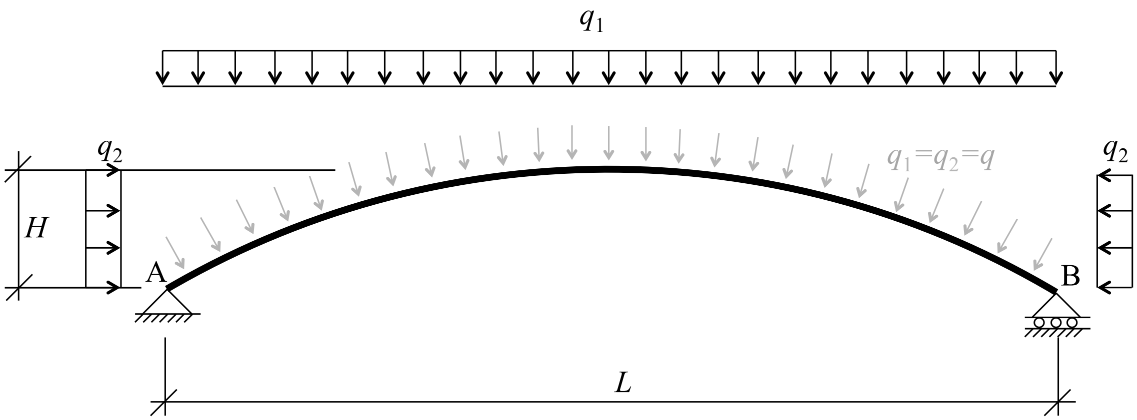
Arch Buckling

Another example on this webpage considers a curved beam; below we first repeat some of the calculations for that example. However, we now avoid giving a load value in order to find its buckling value, and the load is also acting downwards here, in contrast to the other example:



Input values in kN and m

```
E = 200 000 000;
r = 0.1;
L = 10;
H = 1.1;
q1 = q;
q2 = q;
```

Geometry and cross-section calculations

Please see the example involving a curved beam for further details about the following, which is copied from that example:

$$R = \frac{H}{2} + \frac{L^2}{8\,H};$$

$$\theta = 2\,\text{ArcSin}\left[\frac{L}{2\,R}\right];$$

$$\theta 1 = \frac{\pi}{2} - \frac{\theta}{2};$$

$$A = \pi\,r^2;$$

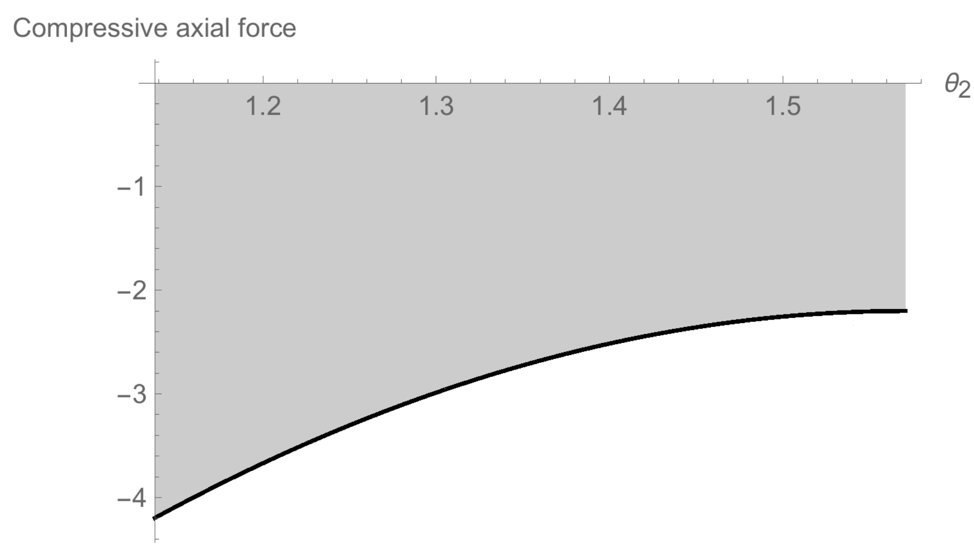
$$I = \frac{\pi}{4} r^4;$$

Internal forces due to external load

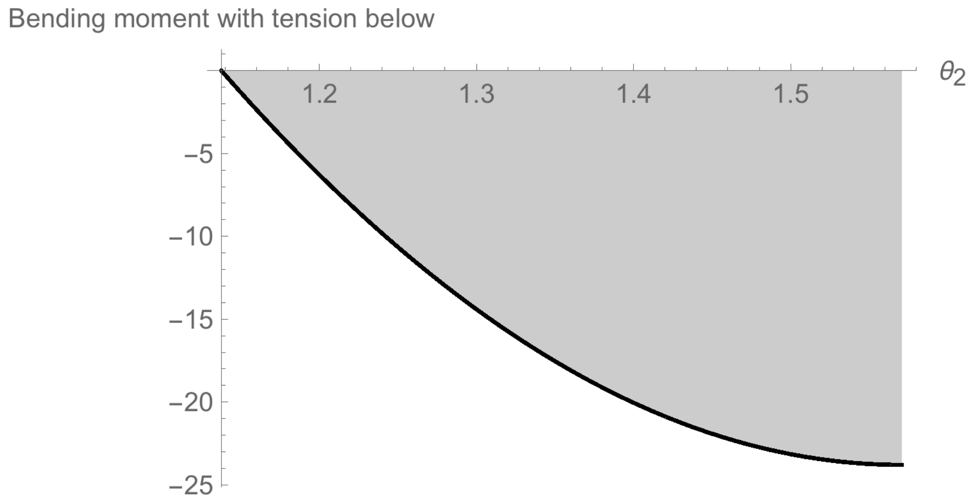
Please see the example involving a curved beam for further details about the following, which is copied from that example:

$$\text{reaction} = \frac{q1\, L}{2};$$

```
Nsupport = -reaction Cos[θ1];

upwardForce = -reaction + q1 x;
rightwardForce = -q2 y;

x = R (Cos[θ1] - Cos[θ2]);
y = R (Sin[θ2] - Sin[θ1]);

axialForce = rightwardForce Sin[θ2] + upwardForce Cos[θ2];
```



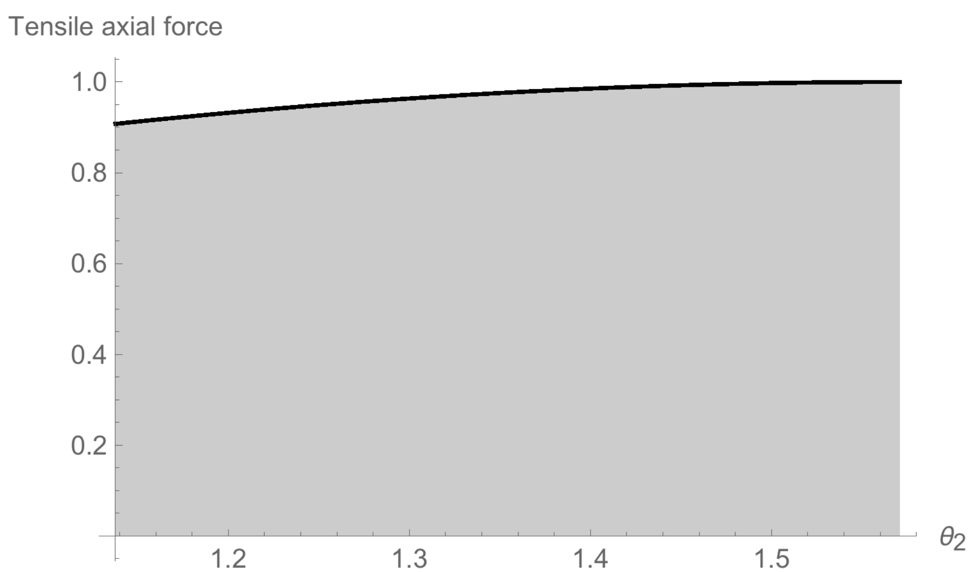
$$\text{bendingMoment} = \frac{q1\, x^2}{2} + \frac{q2\, y^2}{2} - \text{reaction}\, x;$$



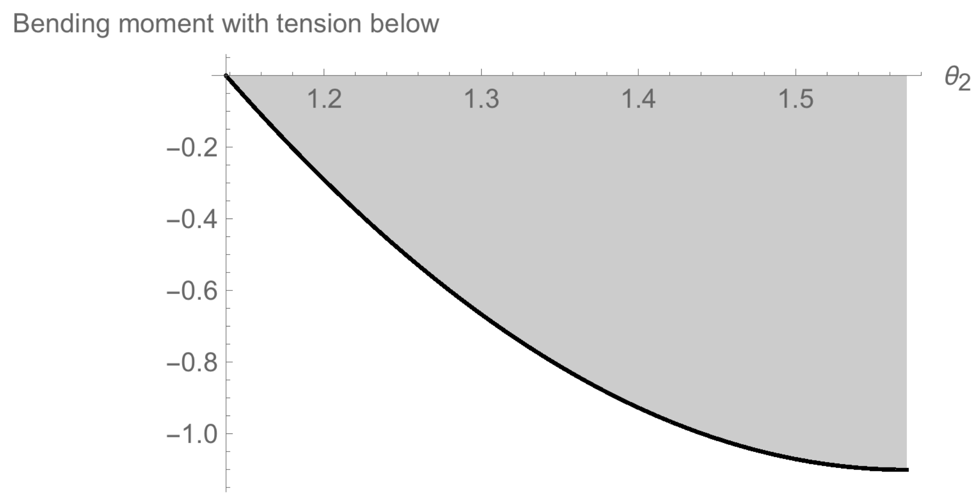
Internal forces due to unit force at B

Please see the example involving a curved beam for further details about the following, which is copied from that example:

```
virtualAxialForceB = Sin[θ2];
```




```
virtualMomentB = -y;
```



Flexibility method to lock support at B

Please see the example involving a curved beam for further details about the following, which is copied from that example:

$$\triangle\text{BFlexure} = 2\int_{\theta 1}^{\frac{\pi}{2}} \left(\text{virtualMomentB}\,\frac{\text{bendingMoment}}{\text{E I}}\right) \text{R}\, d\theta 2;$$

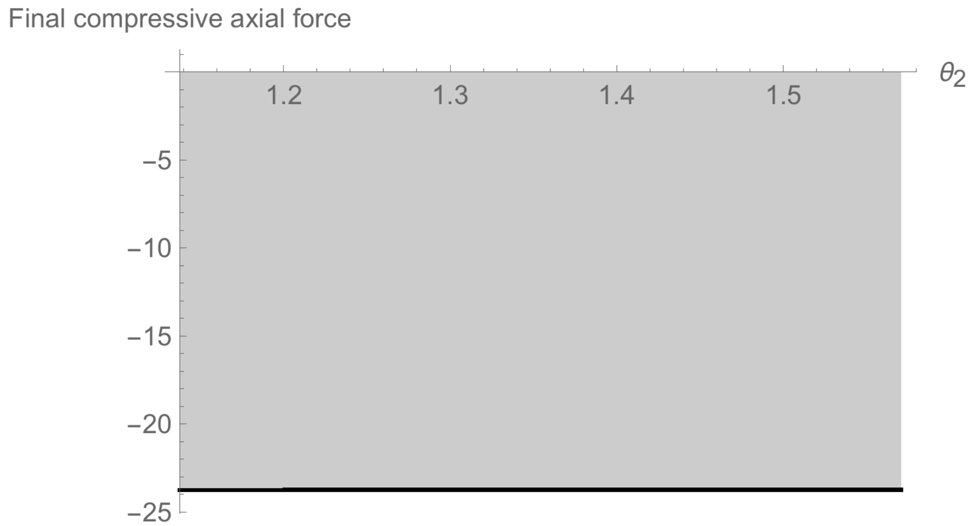
$$\triangle\text{BAxial} = 2\int_{\theta 1}^{\frac{\pi}{2}} \left(\text{virtualAxialForceB}\,\frac{\text{axialForce}}{\text{E A}}\right) \text{R}\, d\theta 2;$$

$$\triangle\text{BB} = 2\int_{\theta 1}^{\frac{\pi}{2}} \left(\frac{\text{virtualMomentB}^2}{\text{E I}}\right) \text{R}\, d\theta 2 + 2\int_{\theta 1}^{\frac{\pi}{2}} \left(\frac{\text{virtualAxialForceB}^2}{\text{E A}}\right) \text{R}\, d\theta 2;$$

$$\triangle\text{B0} = \triangle\text{BFlexure} + \triangle\text{BAxial};$$

$$\text{XB} = -\frac{\triangle\text{B0}}{\triangle\text{BB}};$$

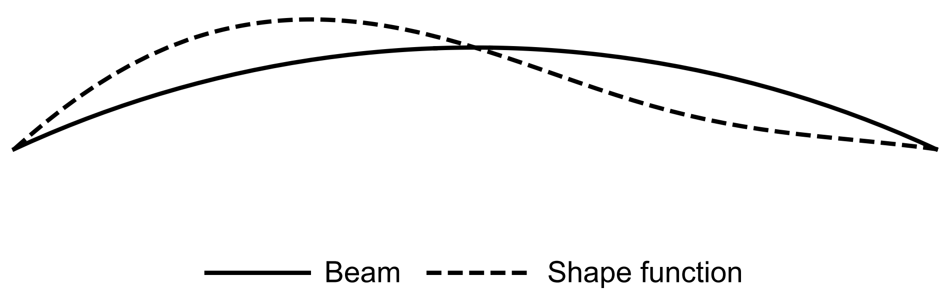
$$\text{finalAxialForce} = \text{axialForce} + \text{virtualAxialForceB XB};$$



Buckling load by the Rayleigh-Ritz energy method

The shape function for Rayleigh-Ritz is has one half sine-wave on each side of the midspan apex:

```
w = C1 Sin[2π/Θ (θ2 - π/2)];
```



The first-order derivative of the shape function is as follows, when we remember that $\frac{dw}{ds} = \frac{dw}{d\theta_2}\frac{d\theta_2}{ds} = \frac{dw}{d\theta_2}\frac{1}{R}$ because ds=Rdθ:

```
dw = 1/R ∂θ2 w;
```

Similarly, the second-order derivative of the shape function is:

```
ddw = 1/R ∂θ2 dw;
```

Substitution into the total potential energy yields

$$\Pi = 2\int_{\theta 1}^{\frac{\pi}{2}} \frac{1}{2}\, E\, I\, \text{ddw}^2\, R\, d\theta 2 - 2\int_{\theta 1}^{\frac{\pi}{2}} \frac{1}{2}\, (-\text{finalAxialForce})\, \text{dw}^2\, R\, d\theta 2;$$

To achieve stationary functional the derivative with respect to C_1 must vanish:

$$q_{\text{crEnergy}} = \text{Solve}[\partial_{C1}\Pi == 0, q]$$

which yields: $\{\{q \to 490.553\}\}$

According to Timoshenko and Gere, the exact solution is:

$$q_{\text{crExact}} = \frac{E\, I}{R^3}\left(\frac{\pi^2}{\left(\frac{\theta}{2}\right)^2} - 1\right)$$

which yields: 479.486