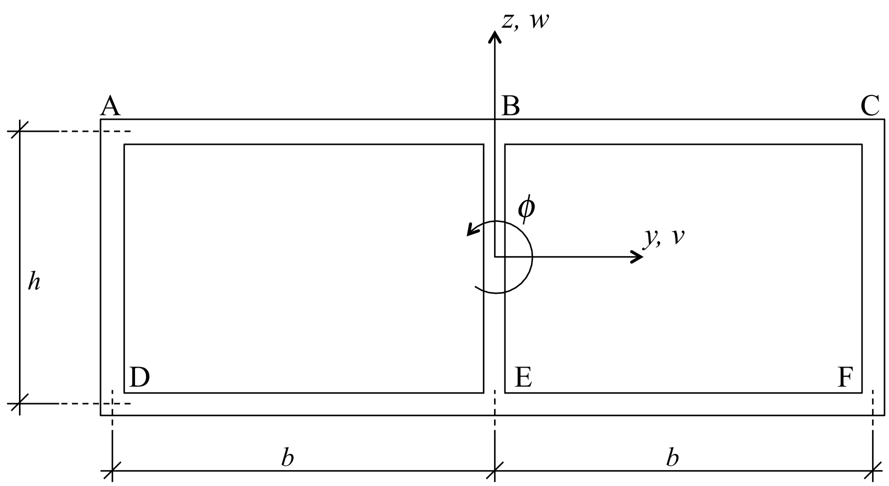
Two-cell cross-section

The cross-section shown in the figure below is considered with the objective of calculating all cross-section constants and determine stresses due to given stress resultants. The cross-section is "thin-walled" and the dimensions *b* and *h* are given to the centre-lines of the cross-section parts. All parts of the cross-section has the thickness *t*.

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b = 300;
h = 200;
t = 10;
```

Identify the value of the cross-section constants A, Iy, Iz, Avy, Avz, J, and Cw and determine all axial and shear stresses caused by the stress resultants given below. When presenting the results in hand calculations, please show how the stresses are distributed over the cross-section, and also how it is distributed over the thickness of the cross-section at key locations.

```
N = 50 000;
My = 50 000 000;
Mz = 5 000 000;
Vz = 50 000;
Vy = 50 000;
TStV = 500 000;
B = 500 000 000;
```

Axial stress due to N

$A = 4\,b\,t + 4\,h\,t$

which yields: 20 000

$$\text{sigmaN} = \frac{N}{A} \text{ // N}$$

which yields: 2.5

Axial stress due My

$$\text{Iy} = 3\left(\frac{t\,h^3}{12}\right) + 4\,b\,t\left(\frac{h}{2}\right)^2 \text{ // N}$$

which yields: 1.4×10^8

$$\text{sigmaMy} = \frac{\text{My}}{\text{Iy}}\frac{h}{2}$$

which yields: 35.7143

Shear stress due to Vz

This is a closed cross-section so the shear flow is statically indeterminate, i.e., it cannot be determined by equilibrium alone. In fact, because there are two cells the degree of indeterminacy is two. Here we arbitrarily select to make cuts at A and C to create an open, statically determinate, cross-section. The unknown shear flow at A and C are the redundants, which are determined by compatibility equations. Because of symmetry, these two redundants are EQUAL in this problem, hence only one unknown appears in the following calculations. That redundant shear flow is called qC and the corresponding value of the first moment of area is QC.

APPROACH 1

One way of determining the redundant is to integrate the Q-diagram (first moment of area) for the statically determinate (cut) cross-section:

$$\text{qC} = -\frac{\text{Vmax}}{\text{Iy}}\frac{\int \frac{Q}{Gt}\,ds}{\int \frac{1}{Gt}\,ds} = -\frac{\text{Vmax}}{\text{Iy}}\text{QC}$$

These integrals are are carried out around each cell, and because G and t are constant we get:

$$\text{QC} = -\frac{\int Q\,ds}{\int ds}$$

The integral in the denominator is simple:

```
sumDs = 2 b + 2 h
```

which yields: 1000

The integral in the numerator is more interesting. The first moment of area, Q, for the open cross-section has these values:

$$\text{QBside} = b\,t\left(h - \frac{h}{2}\right)$$

which yields: 300 000

$$\text{QBbelow} = 2\ \text{QBside}$$

which yields: 600 000

$$\text{QNAmiddle} = \text{QBbelow} + \frac{1}{2}\left(\frac{h}{2}\right)^2 t$$

which yields: 650 000

$$\text{QNAside} = -\frac{h}{2}\,\frac{h}{4}\,t$$

which yields: −50 000

Integration of Q around the LEFT cell yields (adding the redundant in the neighbouring cell in the wall that separates the cells):

$$\text{solution} = \text{Solve}\Big[\text{QCmethod1} == \frac{-1}{\text{sumDs}}\left(\frac{1}{2}\,b\ \text{QBside} + h\ \text{QBbelow} + \frac{2}{3}\,h\ (\text{QNAmiddle} - \text{QBbelow}) + \frac{1}{2}\,b\ \text{QBside} + \frac{2}{3}\,h\ \text{QNAside} + h\ \text{QCmethod1}\right),\ \text{QCmethod1}\Big]$$

which yields: {{QCmethod1 → −175 000}}

which is extracted as follows in *Mathematica*:

```
QC = QCmethod1 /. solution[[1]]
```

which yields: −175 000

It is perhaps interesting to note that the value of the redundant that would have to be obtained for the shear flow to be equal in all three webs (seen from Qdet) is:

$$\frac{2}{3} b\, t\, \frac{h}{2}$$

which yields: 200 000

The fact that the actual solution is somewhat smaller means that more shear flow is attracted to the middle web. However, the smaller value of the width b, the smaller difference in shear flow between the inner web and the outer ones.

APPROACH 2

Another approach is to use the equation that includes the "g-function:"

$$qC = \frac{Vmax}{Iy} \int g\, z\, t\, ds = \frac{Vmax}{Iy} QC$$

where the integrals is made around each respective cell. For simplicity in the following, the integrals are evaluated without the factor Vmax/Iy, thus yielding the value of the first moment of area at A and C:

$$QC = \int g\, z\, t\, ds$$

To normalize the g-function we need the sum of ds/t around both cells:

$$gNorm = 2\,\frac{h}{t} + 2\,\frac{b}{t}$$

which yields: 100

The normalized g-values are then, for different locations around the LEFT CELL, starting at A:

$$gB = 0 + \frac{b}{t}\,\frac{1}{gNorm}\; //\; N$$

which yields: 0.3

$$gE = gB + \frac{h}{t}\,\frac{1}{gNorm}\; //\; N$$

which yields: 0.5

$$gD = gE + \frac{b}{t}\frac{1}{gNorm} // N$$

which yields: 0.8

$$gA = gD + \frac{h}{t}\frac{1}{gNorm} // N$$

which yields: 1.

Next, formulas for g and z are established for each cross-section part, again around the LEFT CELL:

$$gAB = \frac{gB}{b} s;$$
$$zAB = \frac{h}{2};$$

$$gBE = gB + \frac{gE - gB}{h} s;$$
$$zBE = \frac{h}{2} - s;$$

$$gED = gE + \frac{gD - gE}{b} s;$$
$$zED = -\frac{h}{2};$$

$$gDA = gD + \frac{gA - gD}{h} s;$$
$$zDA = -\frac{h}{2} + s;$$

When evaluating the compatibility integral around the LEFT cell, the RIGHT cell will contribute with shear flow at B and C, and vice versa when integrating around the other cell. Those contributions are similar to those in the one-cell cross-section with free flanges:

$$QinflowAtBfromCB = t\, b \left(\frac{h}{2}\right) + QCmethod2;$$
$$QinflowAtEfromCFE = -t\, b \frac{h}{2} - QCmethod2;$$

Then the integral around Cell 1 is:

$$\text{Solve}\left[\text{QCmethod2} == \int_0^b \text{gAB zAB t}\, ds + (\text{gB})\,(\text{QinflowAtBfromCB}) + \int_0^h \text{gBE zBE t}\, ds + (\text{gE})\,(\text{QinflowAtEfromCFE}) + \int_0^b \text{gED zED t}\, ds + \int_0^h \text{gDA zDA t}\, ds,\ \text{QCmethod2}\right]$$

which yields: {{QCmethod2 → −175 000.}}

That result is the same as earlier. Final values of Q at other locations:

$$\text{QBsideFinal} = \text{QBside} + \text{QC}$$

which yields: 125 000

$$\text{QBbelowFinal} = 2\ \text{QBsideFinal}$$

which yields: 250 000

$$\text{QNAmiddleFinal} = \text{QBbelowFinal} + \frac{1}{2}\left(\frac{h}{2}\right)^2 t$$

which yields: 300 000

$$\text{QNAsideFinal} = \text{QC} + \text{QNAside}$$

which yields: −225 000

Final values of shear stress at those locations:

$$\text{tauBside} = \frac{\text{Vz}}{\text{Iy t}}\ \text{QBsideFinal}$$

which yields: 4.46429

$$\text{tauBbelow} = \frac{\text{Vz}}{\text{Iy t}}\ \text{QBbelowFinal}$$

which yields: 8.92857

$$\text{tauNAmiddle} = \frac{\text{Vz}}{\text{Iy t}}\ \text{QNAmiddleFinal}$$

which yields: 10.7143

$$\text{tauNAside} = \frac{\text{Vz}}{\text{Iy t}}\text{QNAsideFinal}$$

which yields: -8.03571

$$\text{tauA} = \frac{\text{Vz}}{\text{Iy t}}\text{QC}$$

which yields: -6.25