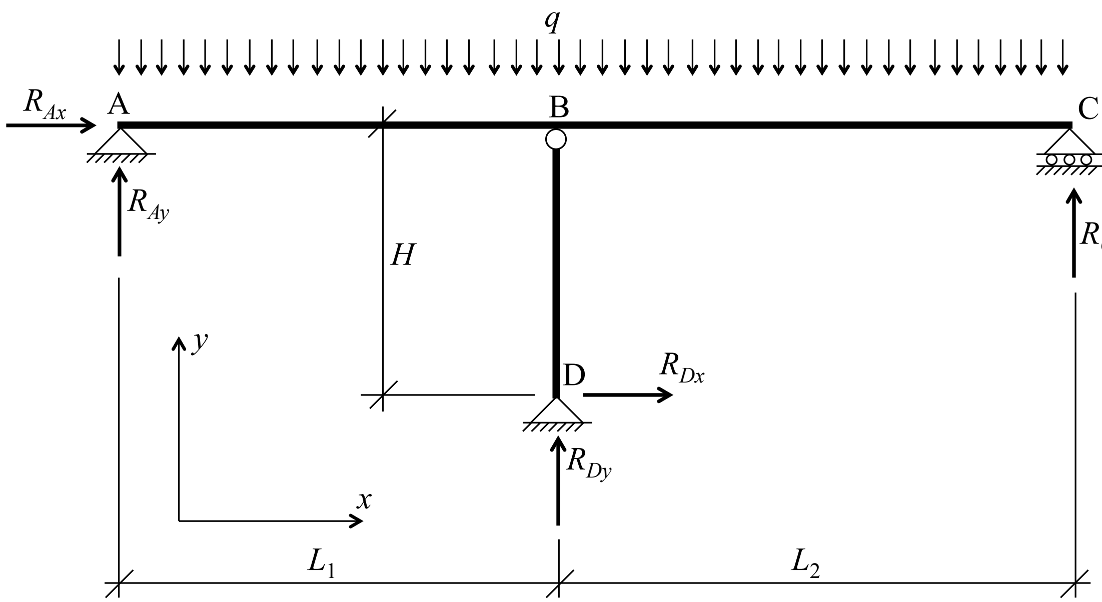
T frame

Consider the frame shown in the figure below with the aim of calculating the bending moment diagram (BMD), shear force diagram (SFD), and axial force diagram (AFD). There is a hinge at B that prevents moment transfer from the column BD to the continuous beam AC.



Input values

Load:

```
q = 50 kN/m;
```

Frame dimensions:

```
L1 = 7 m;
L2 = 10 m;
H = 5 m;
```

Material and cross-section parameters for square reinforced concrete cross-section with dimension *b*:

```
E = 60000 N/mm^2;
b = 0.7 m;
```

$$A = b^2$$

which yields: $0.49\ m^2$

$$I = \frac{b^4}{12}$$

which yields: $0.0200083\ m^4$

Degree of static indeterminacy

The first step in the analysis of any truss or frame structure is to calculate the degree of static indetemincy, DSI. Here we use a formula that employs these symbols:

f = number of unknown **f**orces in each member
m = number of **m**embers
r = number of support **r**eactions
e = number of equilibrium **e**quations at each joint
j = number of **j**oints
h = number of **h**inges, i.e., releases of internal forces

This structure has only frame members, and counting yields:

```
f = 3;
m = 3;
r = 5;
e = 3;
j = 4;
h = 1;
```

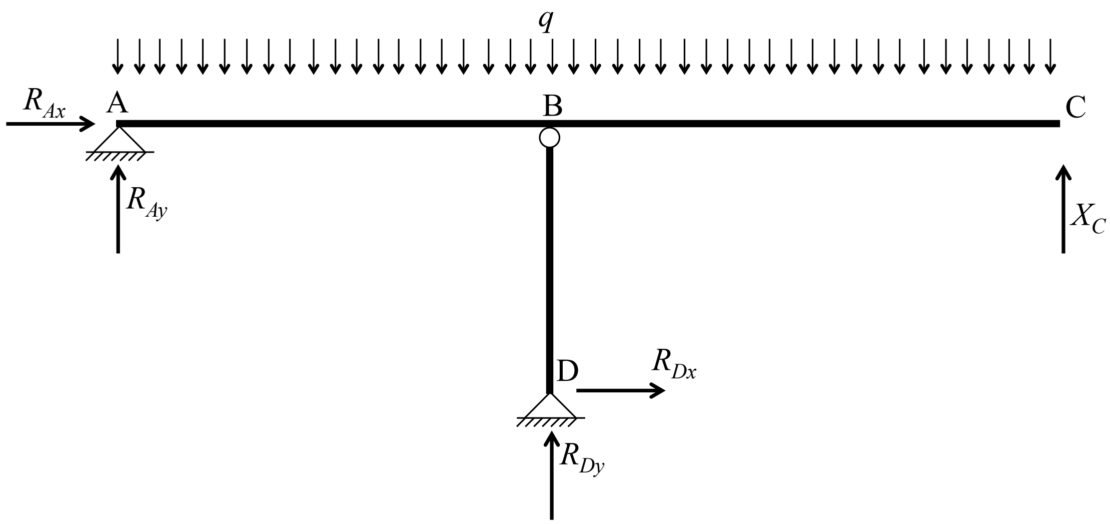
The degree of static indeterminacy is:

$$DSI = (f\,m + r) - (e\,j + h)$$

which yields: 1

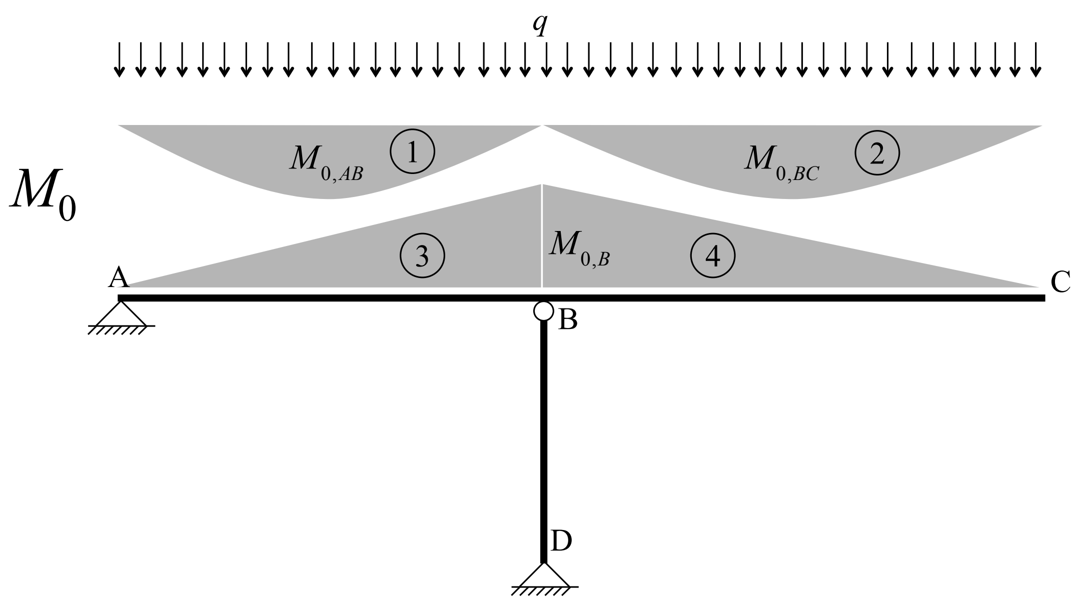
Flexibility method

Because the structure is statically indeterminate we cannot determine the internal forces by equilibrium equations alone. Here the flexibility method is selected. We must select one redundant because DSI=1, and the reaction force at C, R_{Cy}, is used. That means we spend some time analyzing the determinate structure below, without the support at C, either with the load q or a unit force at C:



INTERNAL FORCES DUE TO q

Because the "cut"structure is statically determinate, the values in the BMP shown below are calculated by equilibrium. The separation of the BMD into triangles and parabolas is convenient for the later use of "quick integration formulas" in the virtual work method.



$$\text{M0inAB} = \frac{q\,\text{L1}^2}{8} \quad // \text{ N}$$

which yields: 306.25 m kN

$$\text{M0inBC} = \frac{q\,L2^2}{8} \;//\; N$$

which yields: 625. m kN

$$\text{M0atB} = \frac{q\,L2^2}{2} \;//\; N$$

which yields: 2500. m kN

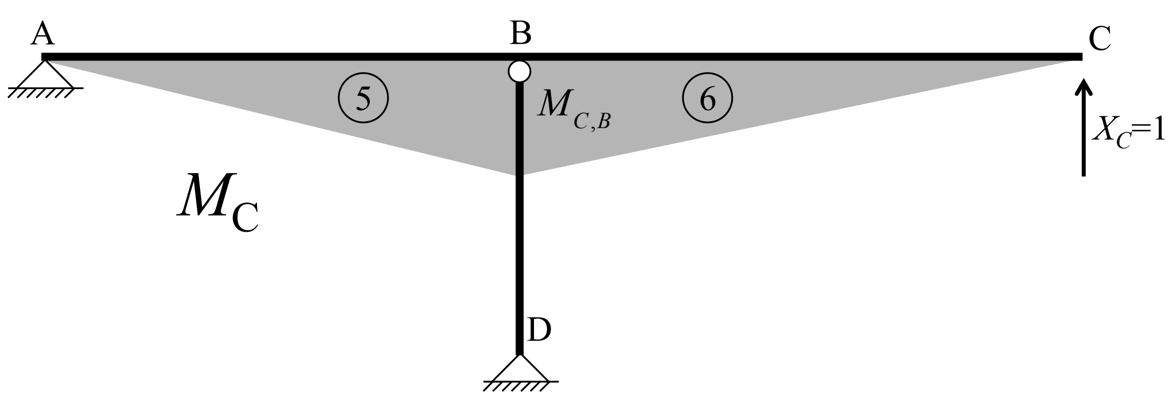
The axial force in the column is calculated by taking moment equilibrium about A. The minus-sign is included because compression is negative:

$$\text{N0} = -q\,(L1 + L2)\,\frac{L1 + L2}{2}\,\frac{1}{L1} \;//\; N$$

which yields: −1032.14 kN

INTERNAL FORCES DUE TO X_C=1

The BMD due to a unit load at C is shown here, with values calculated below. The bending moment has unit "kNm per kN force at C" and the axial force "kN per kN force at C."



$$\text{MCatB} = L2$$

which yields: 10 m

The axial force in column BD is now tension:

$$\text{NC} = (L1 + L2)\,\frac{1}{L1} \;//\; N$$

which yields: 2.42857

COMPATIBILITY EQUATION

The virtual work method is now employed, using "quick integration formulas," to determine the displacement needed by the compatibility equation. The numbers used in the following symbols are circled in the BMD-figures above.

$$\Delta C0from1and5 = \frac{1}{3\,E\,I}\,(M0inAB\,MCatB)\,L1$$

which yields: $0.00595238\,m$

$$\Delta C0from3and5 = -\frac{1}{3\,E\,I}\,(M0atB\,MCatB)\,L1$$

which yields: $-0.0485909\,m$

$$\Delta C0from2and6 = \frac{1}{3\,E\,I}\,(M0inBC\,MCatB)\,L2$$

which yields: $0.0173539\,m$

$$\Delta C0from4and6 = -\frac{1}{3\,E\,I}\,(M0atB\,MCatB)\,L2$$

which yields: $-0.0694155\,m$

$$\Delta C0fromAxial = NC\,\frac{N0}{E\,A}\,H$$

which yields: $-0.000426298\,m$

$$\Delta C0 = \Delta C0from1and5 + \Delta C0from3and5 + \Delta C0from2and6 + \Delta C0from4and6 + \Delta C0fromAxial$$

which yields: $-0.0951264\,m$

The displacement Δ_{CC} has the unit "displacement per unit force at C:"

$$\Delta CCfromBending = \frac{1}{3\,E\,I}\,MCatB^2\,(L1 + L2)$$

which yields: $4.72026 \times 10^{-7}\,m/N$

```
ΔCCfromAxial = NC (NC/(E A)) H
```

which yields: 1.00305×10^{-9} m/N

```
ΔCC = ΔCCfromBending + ΔCCfromAxial
```

which yields: 4.73029×10^{-7} m/N

Solve the compatibility equation:

```
XC = - ΔC0/ΔCC
```

which yields: 201 101. N

FINAL BMD

The final bending moment at B gives tension at the top:

```
MfinalAtB = M0atB - XC MCatB;
UnitConvert[MfinalAtB, "kN metre"]
```

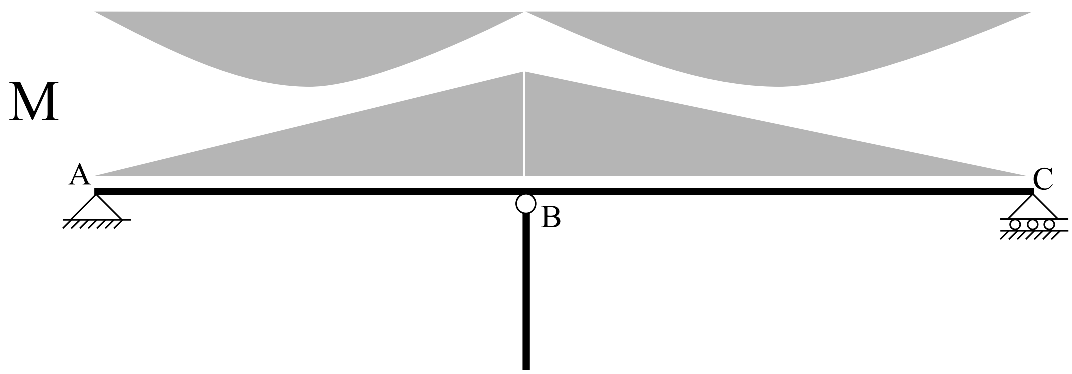
which yields: 488.992 m kN

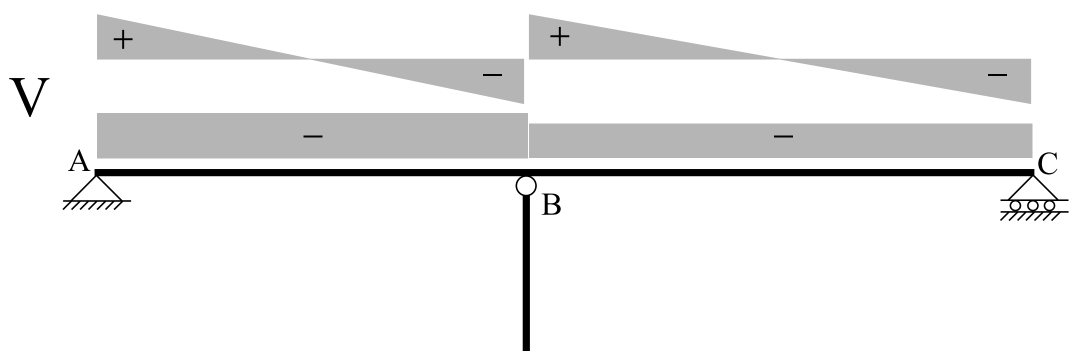
Bending moment at mid-span of AB, using tension at the bottom as positive in this calculation:

```
MfinalInAB = - M0atB/2 + M0inAB + XC MCatB/2;
UnitConvert[MfinalInAB, "kN metre"]
```

which yields: 61.7539 m kN

SFD from BMD





MAXIMUM FIELD MOMENT

To determine the maximum bending moment in span AB we first find the location where the shear is zero. The shear force at A is:

$$VA = \frac{q\,L1}{2} - \frac{MfinalAtB}{L1}$$

which yields: 105 144. N

The change in shear force per unit length is q, which means that the distance from A to the point of zero shear is:

$$distance = \frac{VA}{q}$$

which yields: 2.10288 m

The area of the SFD between A and the point of zero shear equals the change in the bending moment between those two points. Hence, the maximum bending moment in span AB is:

```
MmaxAB = 1/2 VA distance;
UnitConvert[MmaxAB, "kN metre"]
```

which yields: 110.553 m kN

FINAL AXIAL FORCE

The final axial force in column BC is:

```
Nfinal = N0 + XC NC;
UnitConvert[Nfinal, "kN"]
```

which yields: -543.755 kN

Settlement

It is observed that the uniformly distributed load causes bending moment with tension at the top at B. However, the engineers are concerned about the possibility of bending moment with tension at the bottom at B due to settlement of the support at D. Here we use the flexibility method, again with the reaction force at C as redundant, to determine how much downwards settlement there would have to be at D to get a bending moment with tension at the bottom at B while the uniformly distributed load is acting. To accomplish this, we calculate the internal forces due to an unknown settlement. The starting point is to use virtual work to re-calculate ΔC0 due to a settlement at D. The reaction force at D (when the unit force is acting upwards at C) is downwards, same direction as the settlement, hence the minus sign when the settlement work term is moved to the right-hand side:

```
ΔC0settlement = -NC theSettlement
```

which yields: -2.42857 theSettlement

Solve the compatibility equation:

```
XCsettlement = -ΔC0settlement/ΔCC
```

which yields: theSettlement $\left(5.13409 \times 10^{6}\ \text{N/m}\right)$

Calculate the bending moment at B due to the settlement, with tension at the bottom positive:

```
MBsettlement = XCsettlement MCatB
```

which yields: `theSettlement (5.13409 × 10^7 N)`

And finally solve for the settlement at D that would give zero moment at B:

```
Solve[MfinalAtB == MBsettlement, theSettlement]
```

which yields: `{{theSettlement → 0.00952442 m}}`

It is interesting that such a small settlement has a large effect on the bending moment at B.