Stress Concentrations

A fundamental topic in fatigue is the development of cracks in steel. For structures subjected to cyclic loads, such cracks can develop over time and ultimately reach the size at which some dramatic failure occur. While this problem is rare in structures made of "mild steel," the use of "high tensile" steel changes that situation. It is straightforward to double the strength of mild steel by adding carbon, but this may increase the material's brittleness by a factor of fifteen (Gordon 1978). If the stress in the structure was also doubled to take advantage of the increased strength, then the capacity of the material to cope with cracks is reduced by a factor of as much as $2 \cdot 2 \cdot 15=60$, as explained by the equations in this document. This is a dramatic reduction which may make cracks the governing design concern.

The unsuspecting steel engineer may think that the stresses calculated by the theory of elasticity for beams and plates can simply be compared with the yield stress to tell if the structure will stand or fall. However, two additional concerns must often be addressed. One is related to compressive stresses, where the stability against buckling must be considered. The other is related to tensile stresses and is addressed in this document. The ultimate cause of failure is severe cracks, but the underlying cause is stress concentrations. Specifically, the stresses computed by the theory of elasticity increase multi-fold at sharp corners, initial cracks in the material, and other discontinuities. These high and localized stresses can cause cracks to form and grow in an uncontrolled fashion.

The stress increase in the vicinity of some discontinuity is usually expressed by a "stress concentration factor." This factor is multiplied by the average tensile stress in the surrounding area to obtain the peak stress. The stress concentration factor for a crack of length L and tip-radius r is (Inglis 1913):

$$K = 1 + 2 \cdot \sqrt{\frac{L}{r}} \tag{1}$$

It is observed that when $L=r$, i.e., when the crack is semi-circular, the stress concentration factor is 3. Sharper cracks leads to higher stress concentration.

[More cases to come here.]

References

Gordon, J. E. (1978). *Structures: Or Why Things Don't Fall Down*. Penguin Books.

Inglis, C. (1913). "Stresses in a plate due to the presence of cracks and sharp corners." *Transactions of the Royal Institute of Naval Architects*, 60, 219–241.