S-N Diagrams and Miner's Rule

The alternative to fracture mechanics for the consideration of fatigue is the use of S-N curves and an assumption called Miner's rule. The S-N diagrams simply display the number of load cycles, N, at a constant stress amplitude, S, it took for a test-specimen to fail in fatigue. A schematic S-N diagram is shown in Figure 1. It is observed that the horizontal distance to the S-N curve is the "fatigue life" of the test-specimen at a given stress level, and that below a certain stress level there is zero damage accumulation. That stress level is denoted S_∞ and is called the "fatigue limit." The sloped line in Figure 1 is straight because S-N diagrams are presented in a log-log plot. In fact, the SN curve is expressed as

$$S(N) = \left(\frac{K}{N}\right)^{\frac{1}{m}} \tag{1}$$

which implies that

$$\log(N) = \log(K) - m \cdot \log(S) \tag{2}$$

which reveals that m is the negative slope of the S-N curve in the log-log plot while K is another constant that essentially represents the intercept. It is noted from Eq. (2) that the S-N curve can also be written

$$N = \frac{K}{S^m} \tag{3}$$

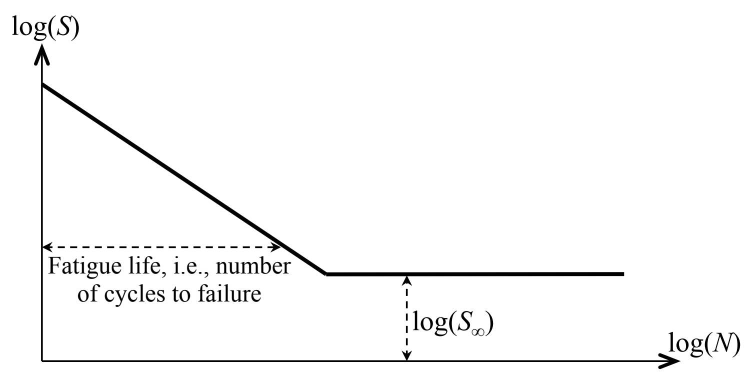

Figure 1: Schematic S-N curve.

The S-N curves described above appear to account only for the stress range and not the average stress. This is not a problem when the stress-cycles oscillates around zero, but could be an issue when the average stress is high, with cycles on top of that. One approach to include the mean stress is the "Goodman correction," which assumes that the damage done by stress cycles with mean S_{mean} and range S_{range} is the same as an auxiliary stress process with zero mean and range S_{aux}, related by the equation (Lutes and Sarkani 1997)

$$\frac{S_{range}}{S_{aux}} + \frac{S_{mean}}{f_u} = 1 \tag{4}$$

where f_u is the ultimate stress capacity of the material. S_{aux} is solved from Eq. (4) and used with the ordinary S-N diagram above. An alternative to the Goodman formulation above is the Gerber formula:

$$\frac{S_{range}}{S_{aux}} + \left(\frac{S_{mean}}{f_u}\right)^2 = 1 \tag{5}$$

Palmgren-Miner's Rule

In practice, S-N diagrams are applied in conjunction with Miner's rule, which states that damage accumulates linearly, in the sense that the fatigue life at different stress levels can be added. As a result, the stress amplitudes that the structure experiences are sorted in bins, where ΔS_i and n_i are the stress range and number of cycles in that stress range, respectively. The total fatigue damage is then measured as

$$D = \sum_{i=1}^{Bins} \frac{n_i}{N_i} \tag{6}$$

Theoretically, $D<1$ means the structure is safe, while $D>1$ implies fatigue failure. In practice, D is not allowed anywhere near unity. Rather, D is typically limited to 0.1 to 0.3, depending on how easy it is to inspect and detect potential damage during the service life of the structure.