Probability of wood defect

In this document probabilities like P(A) and P(A|B) are written P_A and $P_{A,B}$ to facilitate the calculations.

Consider a rare defect with a certain type engineered wood product. For each specimen, define the events D=”defect is present” and I=”indicator shows defect.” During production there is a small probability

$$P_D = 0.008;$$

that any one product comes out defect. However, if a product is defect then there is a

$$P_{I,D} = 0.9;$$

probability that the defect will be detected in a special-purpose test that all products are subjected to. On the other hand, if a product does not have the defect then there is still a

$$P_{I,\overline{D}} = 0.07;$$

probability that the test will categorize it as a defect product. What is the probability that a product that is identified as defect in the test is actually defect?

The sought probability is symbolically written P(D|I) and calculated by Bayes’s theorem:

$$P_{D,I} = \frac{P_{I,D}}{P_I} P_D;$$

The denominator in Bayes is determined by the theorem of total probability:

$$P_I = P_{I,D} P_D + P_{I,\overline{D}} (1 - P_D)$$

which yields: 0.07664

As a result, the probability of defect given that the indicator says yes is:

$$P_{D,I}$$

which yields: 0.0939457

It may appear strange that the chance of a defect is so small even when the indicator says there is a defect present. The reasons for this is the very low probability of defects in the general population of these wood products combined with the imperfection of the measuring device.