Probability of cancelled meeting

Considering an upcoming gathering of four people. This meeting will go ahead if at least two of the participants show up. In other words, the meeting will be cancelled if two or more people are absent. The probability that any given participant will be absent is

```
p = 0.1;
```

and their absences are statistically independent. What is the probability that the meeting will be cancelled?

In the following the events A, B, C, and D are the absence-event for each participant.

Inclusion-exclusion

One way to address the problem is to use the inclusion-inclusion rule. However, caution must be applied. It is relatively straightforward to calculate the probability that ONE OR MORE of the participants are absent:

P(A)+P(B)+P(C)+P(D)
-P(AB)-P(AC)-P(AD)-P(BC)-P(BD)-P(CD)
+P(ABC)+P(ABD)+P(ACD)+P(BCD)
-P(ABCD)

That probability is:

$$4p - 6p^2 + 4p^3 - p^4$$

which yields: 0.3439

However, when asking for the probability that TWO OR MORE of the participants are absent it gets more complicated to count the overlapping events in the Venn diagram. If we first calculate

P(AB)+P(AC)+P(AD)+P(BC)+P(BD)+P(CD)

then we have actually counted the overlapping areas ABC, ABD, ACD, and BCD three times each. Hence, for each of those four we must substract two for a total of eight overlap areas in the Venn diagram. In that Venn diagram we have now counted the correct number of joint probabilities with two events, and also the correct number of probabilities with three events, but the joint probability with four events, ABCD, appear once in each of the events ABC, ABD, ACD, and BCD. Hence we must now add back three of them for the final answer:

$$6\,p^2 - 8\,p^3 + 3\,p^4$$

which yields: 0.0523

Counting scenarios

The inclusion-exclusion rule is cumbersome when we cannot start at the top level with the basic probabilities for A, B, C, and D. Another approach is to recognize that TWO OR MORE is exactly two or exactly three or exactly four absent participants. Or alternatively that it is not full attendance and not exactly one absent participant. Hence, what we seek in this approach is the probability of ONE AND ONLY ONE, TWO AND ONLY TWO participants absent, etc.

Consider first the probability that NONE ARE ABSENT. Here there is only one scenario to count, with probability

```
PnoneAbsent = (1 - p)^4
```

which yields: 0.6561

Now consider the probability that ONE AND ONLY ONE participant is absent. Using the binomial coefficent $\binom{n}{k}$="n choose k" there are "4 choose 1" scenarios in which that happens:

```
scenariosWithOneAndOnlyOneAbsent = Binomial[4, 1]
```

which yields: 4

Pascal's triangle can be used to determine that value, i.e., the number of ways in which 1 is absent from a meeting of 4 people:

n=0 1
n=1 1 1
n=2 1 2 1
n=3 1 3 3 1
n=4 1 **4** 6 4 1

Hence, the probability that one and only one person is absent is:

```
PoneAndOnlyOneAbsent = scenariosWithOneAndOnlyOneAbsent (1 - p)^3 p
```

which yields: 0.2916

Similarly we obtain the probabilities that TWO AND ONLY TWO participants are absent:

```
scenariosWithTwoAndOnlyTwoAbsent = Binomial[4, 2]
```

which yields: 6

```
PtwoAndOnlyTwoAbsent = scenariosWithTwoAndOnlyTwoAbsent (1 - p)^2 p^2
```

which yields: 0.0486

Similarly we obtain the probabilities that THREE AND ONLY THREE participants are absent:

```
scenariosWithThreeAndOnlyThreeAbsent = Binomial[4, 3]
```

which yields: 4

```
PthreeAndOnlyThreeAbsent = scenariosWithThreeAndOnlyThreeAbsent (1 - p) p^3
```

which yields: 0.0036

And finally the probability that all four participants are absent:

```
PallAbsent = p^4
```

which yields: 0.0001

This gives the probability that TWO OR MORE participants are absent:

```
PtwoAndOnlyTwoAbsent + PthreeAndOnlyThreeAbsent + PallAbsent
```

which yields: 0.0523

or computed another way, using the same same probabilities:

```
1 - PnoneAbsent - PoneAndOnlyOneAbsent
```

which yields: 0.0523

By the way, notice how this approach gives the probability of ONE OR MORE absentees, calculated in the beginning:

```
1 - PnoneAbsent
```

which yields: `0.3439`

Probability tree

Another approach is to set up a "probability tree" of all possible options, starting with the branch separating A being present or not, and for each of those two outcomes B is present or not, etc. At the end of each branch there is a probability, namely the product of the probabilities at the preceeding branches.

System reliability

Yet another approach is to consider the problem a system reliability problem, where the minimum cut sets of the system are the possible combination of two, three, or four people missing the meeting:

c1 = {A, B}
c2 = {A, C}
c3 = {A, D}
c4 = {B, C}
c5 = {B, D}
c6 = {C, D}

This is a series system of six sub-parallel systems and we know the failure probability for each sub-parallel system. The individual component probabilities are:

$$pComp = p^2$$

which yields: `0.01`

The intersection probabilities are:

```
probTable = {{pComp, p^3, p^3, p^3, p^3, p^4}, {p^3, pComp, p^3, p^3, p^4, p^3},
    {p^3, p^3, pComp, p^4, p^3, p^3}, {p^3, p^3, p^4, pComp, p^3, p^3},
    {p^3, p^4, p^3, p^3, pComp, p^3}, {p^4, p^3, p^3, p^3, p^3, pComp}};
probTable // MatrixForm
```

which yields:

$$\begin{pmatrix} 0.01 & 0.001 & 0.001 & 0.001 & 0.001 & 0.0001 \\ 0.001 & 0.01 & 0.001 & 0.001 & 0.0001 & 0.001 \\ 0.001 & 0.001 & 0.01 & 0.0001 & 0.001 & 0.001 \\ 0.001 & 0.001 & 0.0001 & 0.01 & 0.001 & 0.001 \\ 0.001 & 0.0001 & 0.001 & 0.001 & 0.01 & 0.001 \\ 0.0001 & 0.001 & 0.001 & 0.001 & 0.001 & 0.01 \end{pmatrix}$$

Unimodal bounds:

```
pUniLower = pComp
```

which yields: 0.01

```
pUniUpper = Min[1, (6 pComp)]
```

which yields: 0.06

Bimodal bounds require the calculation of correlation between the cut sets, to find the bimodal intersection probabilities:

$$\text{pBiLower} = \text{pComp} + \sum_{m=2}^{6} \text{Max}\left[0, \text{pComp} - \sum_{j=1}^{m-1} \text{probTable}[[m, j]]\right]$$

which yields: 0.0477

$$\text{pBiUpper} = \text{pComp} + \sum_{m=2}^{6} (\text{pComp} - \text{Max}[\text{probTable}[[m, 1 ;; m-1]]])$$

which yields: 0.055

We observe that the correct answer is located between those two bounds.