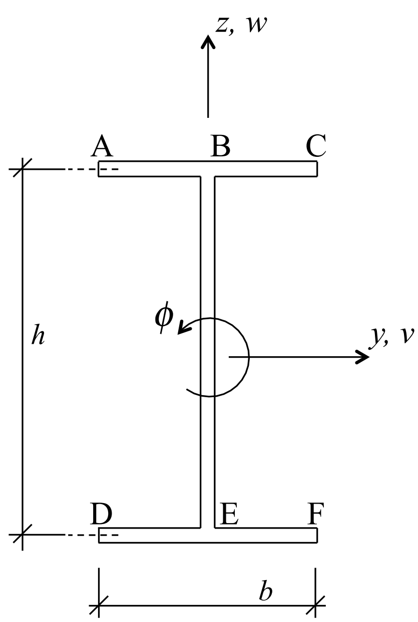
I cross-section

The cross-section shown in the following figure is analyzed here, with the objective of calculating several cross-section contants and also the stresses due to various stress resultants. This is a “thin-walled” cross-section with all parts having the thickess *t*.



Input values

Dimensions:

```
b = 300 mm;
h = 500 mm;
t = 15 mm;
```

Stress resultants:

```
N = 50 kN;
My = 5 m kN;
Mz = 5 m kN;
Vz = 50 kN;
Vy = 50 kN;
TStV = 0.5 m kN;
B = 0.5 kN * m^2;
```

Axial stress due to N

Cross-section area:

$$A = 2\,b\,t + h\,t$$

which yields: $16\,500\ \text{mm}^2$

Axial stress, distributed uniformly over the cross-section:

```
σN = N/A;
UnitConvert[σN, "N/mm2"] // N
```

which yields: $3.0303\ \text{N/mm}^2$

Axial stress due to My

Moment of inertia:

$$Iy = \frac{t\,h^3}{12} + 2\,b\,t\left(\frac{h}{2}\right)^2 \; // \; N$$

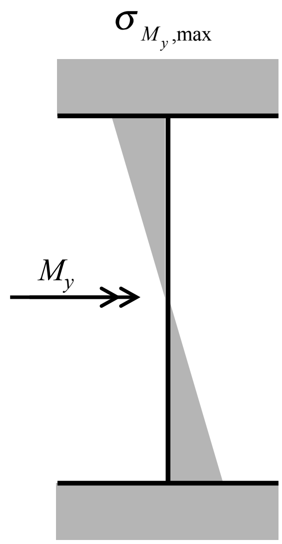
which yields: $7.1875 \times 10^8\ \text{mm}^4$

The maximum axial stress, which appears at the location furthest away from the neutral axis:

```
σMyMax = My/Iy h/2;
UnitConvert[σMyMax, "N/mm2"]
```

which yields: $1.73913\ \text{N/mm}^2$

The stress is distributed over the cross-section as shown in this figure:



Axial stress due to Mz

Moment of inertia:

```
Iz = 2 t b^3 / 12 // N
```

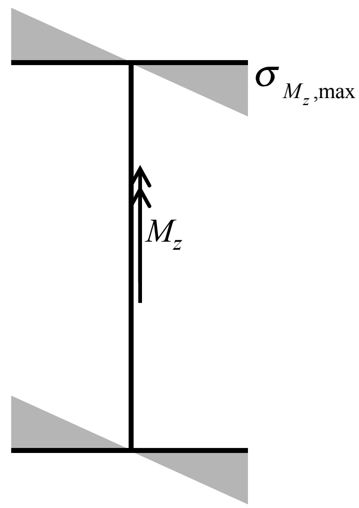
which yields: 6.75×10^7 mm^4

The maximum axial stress, which appears at the location furthest away from the neutral axis:

```
σMzMax = Mz/Iz b/2;
UnitConvert[σMzMax, "N/mm2"]
```

which yields: 11.1111 N/mm^2

This axial stress is distributed as follows over the cross-section:



Shear stress due to Vz

Due to symmetry of the cross-section the shear stress distribution is double-symmetric. As shown in the figure below the shear stress is zero at A and increases linearly towards the following value at B:

$$\tau VzBside = \frac{Vz}{Iy\,t}\, t\, \frac{b}{2}\, \frac{h}{2};$$

```
UnitConvert[τVzBside, "N/mm2"]
```

which yields: $2.6087\ \mathrm{N/mm^2}$

The shear stress immediately below B is twice that value because of the two “rivers” of shear flow meeting at B:

```
τVzBbelow = 2 τVzBside;
UnitConvert[τVzBbelow, "N/mm2"]
```

which yields: $5.21739\ \mathrm{N/mm^2}$

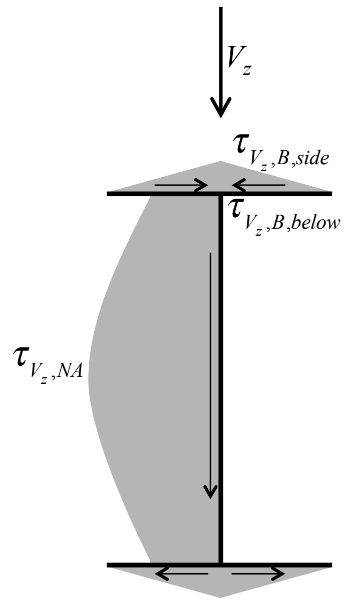
The maximum shear stress, which appears at the neutral axis, is:

$$\tau VzNA = \frac{Vz}{Iy\,t}\left(t\, b\, \frac{h}{2} + t\, \frac{h}{2}\, \frac{h}{4}\right);$$

```
UnitConvert[τVzNA, "N/mm2"]
```

which yields: $7.3913\ \mathrm{N/mm^2}$

The distribution of shear stresses due to Vz looks like this:



Shear area Avz

The formula for the exact shear area, as a fraction of the total area, gives, using "quick integration formulas" and values for the first moment of area at the same locations as the shear stress was calculated:

$$\text{QVzBside} = t\,\frac{b}{2}\,\frac{h}{2};$$

$$\text{QVzG} = t\,b\,\frac{h}{2} + t\,\frac{h}{2}\,\frac{h}{4};$$

$$\beta z = \text{Iy}^2 \Big/ \left(A\left(4\times\frac{1}{3}\left(\frac{\text{QVzBside}}{t}\right)^2 \frac{b}{2}\,t + \left(\frac{2\,\text{QVzBside}}{t}\right)^2 h\,t + \frac{8}{15}\left(\frac{\text{QVzG} - \text{QVzBside}}{t}\right)^2 h\,t\right)\right)$$

which yields: 0.479375

That means the shear area is:

$$\text{Avz} = \beta z\,A$$

which yields: $7909.69\ \text{mm}^2$

Compare that with the approximation of using the area of the parts aligned with the shear force direction:

```
AvzAppx = h t
```

which yields: $7500\ \text{mm}^2$

We observe that the approximate approach underestimates the shear area because we are not counting the flanges, which do indeed take some of the shear flow.

Shear stress due to Vy

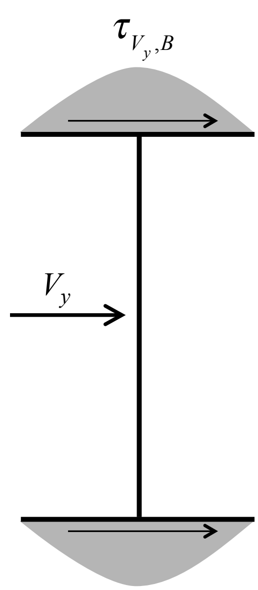
Due to symmetry of the cross-section this shear stress distribution is also double-symmetric. The shear stress is zero at A and increases quadratically towards the following value at B:

$$\tau VyB = \frac{Vy}{Iz\,t}\left(\frac{b}{2}\,t\,\frac{b}{4}\right);$$

```
UnitConvert[τVyB, "N/mm2"]
```

which yields: $8.33333\ \text{N/mm}^2$

The first moment of area everywhere along the web is zero, hence the shear stress in the web is zero. That means the distribution of this shear stress looks like this:



Shear area Avy

The formula for the exact shear area as a fraction of the total area gives, again using "quick integration formulas:"

$$\text{QVyB} = \frac{b}{2}\, t\, \frac{b}{4};$$

$$\beta y = \frac{Iz^2}{A\left(2\ \frac{8}{15}\left(\frac{\text{QVyB}}{t}\right)^2 b\, t\right)}$$

which yields: 0.454545

That means the shear area is:

$$\text{Avz} = \beta z\, A$$

which yields: $7909.69\ \text{mm}^2$

Compare that with the approximation of using the area of the parts aligned with the shear force direction:

$$\text{AvzAppx} = 2\, b\, t$$

which yields: $9000\ \text{mm}^2$

We now see that the approximate approach overestimates the shear area because we are counting the full flanges although they do not have maximum shear flow all over, and the web does not help with any shear flow at all.

Shear stress due to $T_{St.V}$.

The cross-section constant for St. Venant torsion is:

$$J = \frac{1}{3}\,(2\, b + h)\, t^3$$

which yields: $1\,237\,500\ \text{mm}^4$

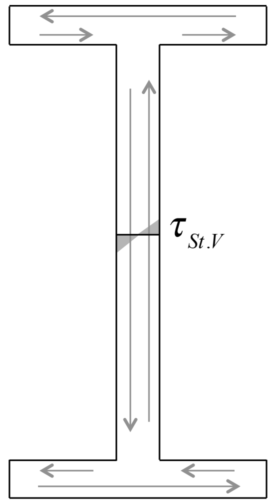
The maximum shear stress, which is constant around all edges is:

$$\tau \text{StV} = \left(\frac{8\ \frac{t}{2}}{t^2}\right)\left(\frac{3\ \text{TStV}}{4\, t\,(2\, b + h)}\right);$$

```
UnitConvert[τStV, "N/mm2"]
```

which yields: $6.06061\ \text{N/mm}^2$

The flow of shear stresses due to Saint Venant torsion looks like this:



Axial stress due to B

Before turning to the omega diagram, a simplified and approximate approach is possible for I-beams, considering the bending moment in each flange:

$$\text{Mflange} = \frac{B}{h}$$

which yields: 1. m kN

The axial stress due to that bending moment is:

$$\sigma\text{Bappx} = \frac{\text{Mflange}}{\left(\frac{t\,b^3}{12}\right)}\,\frac{b}{2};$$

```
UnitConvert[σBappx, "N/mm2"]
```

which yields: $4.44444\ \text{N/mm}^2$

In the approximate approach, the cross-section constant C_ω is defined as:

$$\text{CwAppx} = \left(2\,\frac{t\,b^3}{12}\right)\frac{h^2}{2}$$

which yields: $8\,437\,500\,000\,000\ \text{mm}^6$

Now using the omega diagram: Due to double-symmetry the shear centre is known, hence the final values of the omega diagram can be established directly. It has the following maximum value at A, C, D, and F:

$$\Omega max = \frac{b}{2}\frac{h}{2}$$

which yields: $37\,500\ \text{mm}^2$

The cross-section constant C_w is the integral of the squared omega diagram, which by "quick integration" is:

$$Cw = 4\,t\,\frac{1}{3}\,\Omega max^2\,\frac{b}{2}$$

which yields: $4\,218\,750\,000\,000\ \text{mm}^6$

That gives the same maximum axial stress value as calculated by the "approximate" approach above:

$$\sigma Bmax = \frac{B}{Cw}\,\Omega max;$$

```
UnitConvert[σBmax, "N/mm2"]
```

which yields: $4.44444\ \text{N/mm}^2$

The axial stress is distributed over the cross-section proportional to the omega diagram, which looks like this: