Golden Section Algorithm

This algorithm is implemented in Rts in the class *RGoldenSectionLineSearchAlgorithm*. It is an interesting algorithm to determine the solution to the minimization problem

$$x^* = \arg\min\{F(x)\} \tag{1}$$

based on the golden section ratio, which also appears also in aesthetical studies. At every iteration uses the value of $F(x)$ at three x-values and tests the value of $F(x)$ at a fourth. In Figure 1 the four points are labelled x_i, i=1,2,3,4. Suppose we have evaluated $F(x_1)$, $F(x_2)$, and $F(x_4)$ but not $F(x_3)$. Then $F(x_3)$ is evaluated and its value is below $F(x_2)$, which implies that the solution must lie somewhere in the range x_2 to x_4. Conversely, if $F(x_2) < F(x_3)$ then the solution must lie in the range x_1 to x_3. The interval that contains the solution gets an added point, located according to the golden section ratio as in the initial four points.

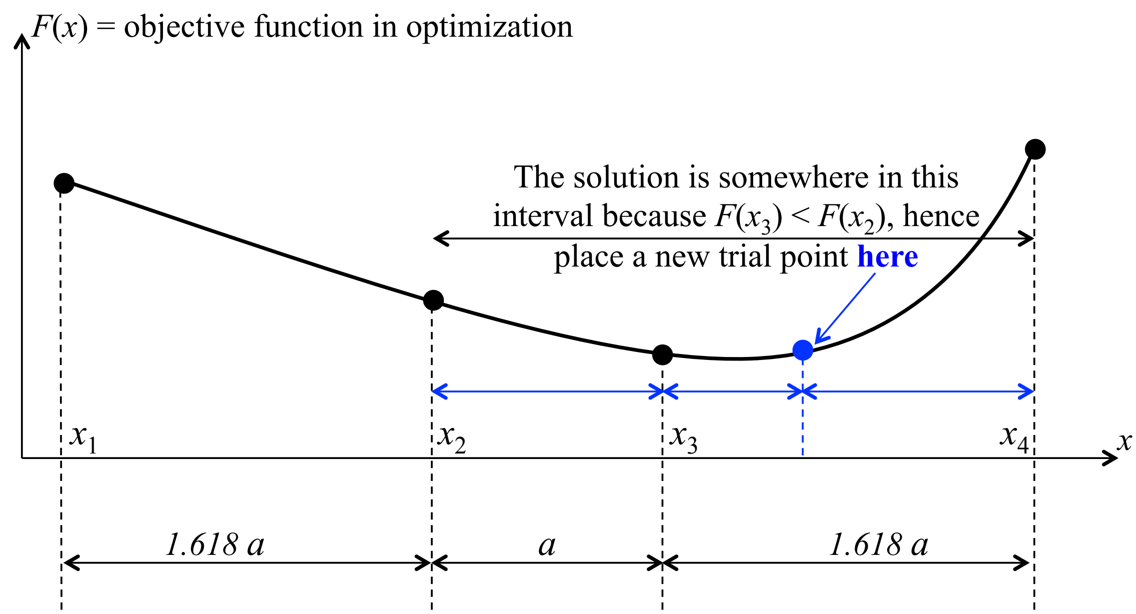

Figure 1: Golden section algorithm.

The ratio of the "large intervals" (say, a) to the "small intervals" (say, b) is determined by the two equations

$$\frac{a}{b} = \frac{a+b}{a} = \varphi \tag{2}$$

where φ is a constant. One approach for determining φ is to write the second equation as

$$\frac{a+b}{a} = 1 + \frac{b}{a} = 1 + \frac{1}{\varphi} = \varphi \tag{3}$$

where the first equation was used in the second equality. The resulting equation is a second-order equation

$$1+\frac{1}{\varphi}=\varphi \quad \Rightarrow \quad \varphi^2-\varphi-1=0 \tag{4}$$

which has the solutions

$$\varphi=\frac{1\pm\sqrt{5}}{2} \tag{5}$$

The negative root is discarded and the positive root is $\varphi \approx 1.618$.