Fracture Mechanics

Fracture mechanics is an analytical approach to the study of crack propagation. In contrast, the subsequent section on fatigue gives an empirical alternative to determine the number of stress cycles that can be handled before a crack grows out of control. The concept of energy is important in fracture mechanics. In particular, the energy needed to grow a crack is compared with the energy provided by the surrounding material. The former is proportional to $W \cdot L$, where W is the "work of fracture" of the material and L is the crack length. Conversely, the energy released around the crack is proportional to L^2, which is understood by considering triangular areas on either side of the crack. These two terms are illustrated in Figure 1 (Gordon 1978). Because the energy needed to grow the crack varies linearly with L and the energy given by the surrounding material varies quadratically with L there will be a crack length at which the crack starts growing uncontrollably. This crack length is denoted L_g in the figure. The subscript on L_g is from Griffith, who developed the following formula for the "critical crack length" (Griffith 1921):

$$L_g = \frac{2 \cdot W \cdot E}{\pi \cdot s^2} \tag{1}$$

where W=work of fracture, E=Young's modulus, and s=average tensile stress in the surrounding area without stress concentration included.

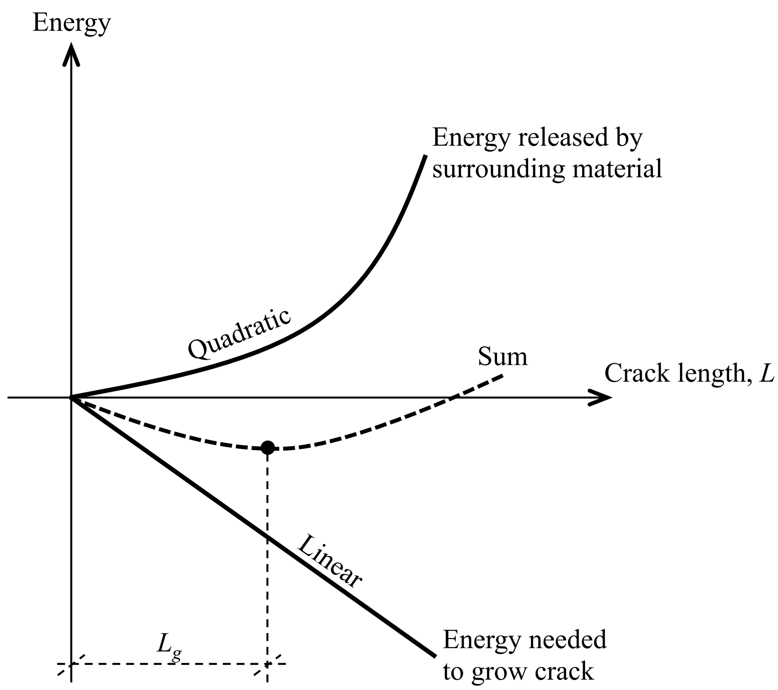

Figure 1: Energy balance in crack growth.

The problem of fatigue, namely the potential growth of cracks due to a high number of load cycles, is an important design concern, particularly for steel structures. Fatigue may occur in many structural applications, from bridges to ships. For example, the number of wave-induced bending cycles of a typical ship may be in the order of 10^8 during a 20-year time period (Hughes and Paik 2010). In addition comes cycles due to loading and unloading, as well as engine and propeller vibration.

Fracture mechanics represents an analytical approach to address fatigue, i.e., crack growth due to cyclic loading. The other approach is to use empirical S-N curves, which is addressed in the next section. In fracture mechanics, the crack growth is formulated by means of a damage accumulation model, such as:

$$\frac{da}{dN} = C \cdot \left(\Delta K \right)^m \tag{2}$$

where a is the crack length, N is number of load cycles, C and m are material constants, and ΔK is the range of the stress intensity factor K, which represents the stress range. Techniques to solve for a are presented in another document on probabilistic damage models, specifically damage accumulation models.

References

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