

# Euclid's Algorithm

Perhaps the oldest algorithm in current use is that described by Euclid to determine the greatest common divisor of two integers. Euclid was a Greek mathematician born in 335BC, the year before Alexander the Great started his ten-year war-campaign to conquer the Persian Empire. On his journey Alexander visited Egypt and gave the architect Deinokrates the task of planning the city of Alexandria at the site of the fishing village Rhakotis, a substantial engineering accomplishment. Euclid ended up living there for 40 years, where he authored "Elements" and thus established the field of geometry.

The task solved by Euclid's algorithm is finding the largest integer, i.e., greatest common divisor, which divides the two integers without leaving a remainder. The algorithm is based on the fact that the greatest common divisor will also divide the *difference* between the two numbers without leaving a remainder. Hence, the algorithm of Euclid proceeds by repeatedly subtracting the small number from the large number until they are either equal, in which case the greatest common divisor is the small number, or until the remainder is smaller than the small number. In the latter case, the remainder is used as the new small number and the previous small number is now the large number. This is repeated until the remainder is zero, at which time the small number is the greatest common divisor. In C++ this reads:

```
double findGreatestCommonDivisor(int a, int b)
{
    // First make sure a is the largest of the two numbers
    if (a<b) {
        double c = a;
        a = b;
        b = c;
    }

    // Then loop until convergence
    bool convergence = false;
    while (!convergence) {

        // Subtract as many multiples of b from a
        // as possible and check the remainder
        double remainder = a%b;
        if (remainder == 0.0) {
            convergence = true;
            return b;
        }
        else {

            // Let b be the new big number, let the remainder be
            // the new small number, and try again
            a = b;
            b = remainder;
        }
    }
}
```