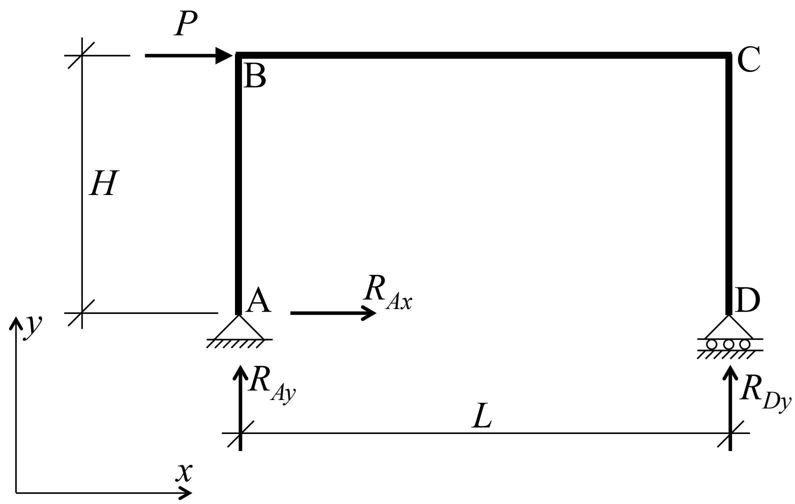
Determinate portal frame

Consider the frame shown in the figure below with the aim of calculating the bending moment diagram (BMD), shear force diagram (SFD), and axial force diagram (AFD).



Input values in kN and m

```
H = 4;
L = 5;
P = 20;
```

Degree of static indeterminacy

The first step in the analysis of any truss or frame structure is to calculate the degree of static indetemincy, DSI. Here we use a formula that employs these symbols:

f = number of unknown **f**orces in each member
m = number of **m**embers
r = number of support **r**eactions
e = number of equilibrium **e**quations at each joint
j = number of **j**oints
h = number of **h**inges, i.e., releases of internal forces

This structure has only frame members, and counting yields:

```
f = 3;
m = 3;
r = 3;
e = 3;
j = 4;
h = 0;
```

The degree of static indeterminacy is:

```
DSI = (f m + r) - (e j + h)
```

which yields: 0

Bending moment value at key locations

Because the structure is statically determinate (DSI=0) we need only equilibrium equations to determine the internal forces, i.e., bending moments, etc. The first step in this process is always to determine the support reactions. That is accomplished with these equilibrium equations:

```
xEquil = P + RAx == 0;
yEquil = RAy + RDy == 0;
rotEquilA = P H - RDy L == 0;
```

We can solve those equations one-by-one, but they are here solved together:

```
reactions = Solve[{xEquil, yEquil, rotEquilA}, {RAx, RAy, RDy}]
```

which yields: {{RAx → -20, RAy → -16, RDy → 16}}

Knowing the reactions we can determine the internal forces anywhere on the structure. Thus, we proceed to determine the bending moment value at key locations (by making a cut at that location) and later we know the variation of the bending moment between those locations by looking at the distributed load. In this case there are no distributed loads, hence all bending moment diagrams vary linearly. Bending moment at B, here considering tension on the left as positive and substituting the reaction values determined above:

```
MB = (RAx H) /. reactions[[1]]
```

which yields: -80

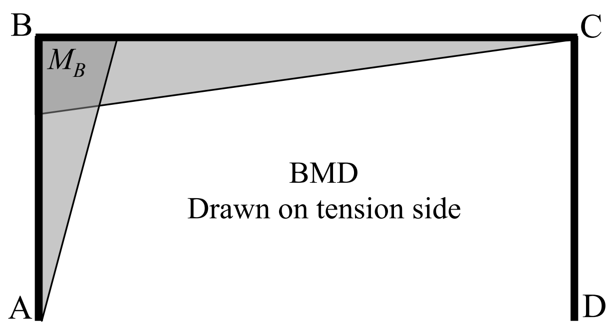
The bending moment at C is zero, even if we consider the forces to the left of C:

```
MC = (RAy L - RAx H) /. reactions[[1]]
```

which yields: 0

BMD from key values

Having those bending moment value we can draw the following BMD; the lines are straight because there is no distributed load on any member:



SFD from BMD

To determine the shear force diagram we consider one member at a time. Because $V = \frac{dM}{dx}$ the SFD is the slope of the BMD. Because we have drawn the BMD on the tension side, the following sign convention applies: downhill slope of the BMD (when we walk from left to right) means positive shear, i.e., clockwise shear. The result is shown in the figure below (it does not matter on which side of the member the diagram is, as long as we include the sign) with the following shear force values:

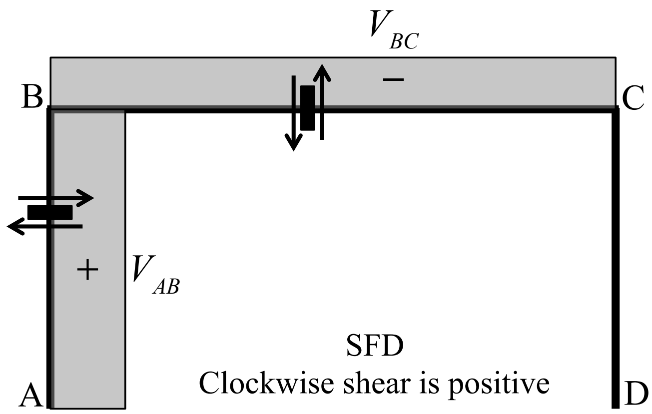
$$VAB = Abs\left[\frac{MB}{H}\right]$$

which yields: 20

$$VBC = Abs\left[\frac{MB}{L}\right]$$

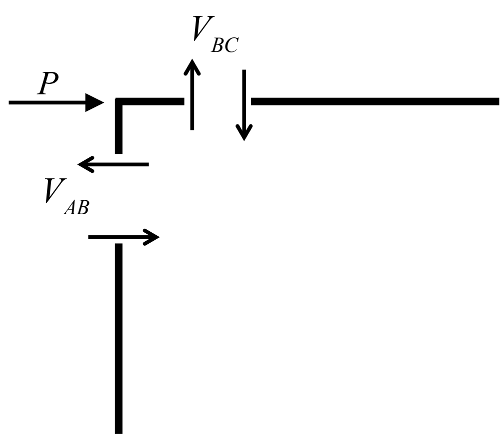
which yields: 16

The diagram looks like this:

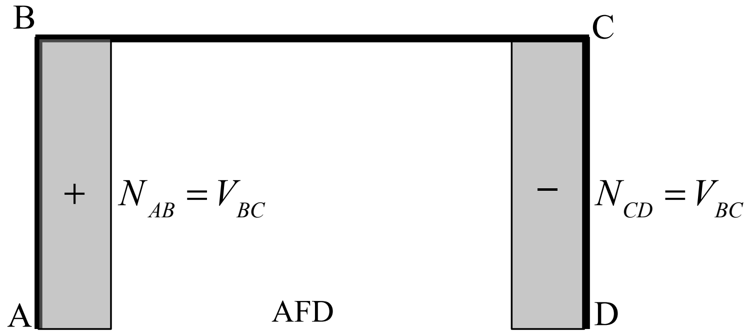


AFD from SFD

The AFD is determined by looking at the SFD at the joints. The SFD implies certain forces there that must be matched by axial forces to maintain equilibrium. Here we could make life a little easier by starting at joints A and D, where the known reaction forces directly give the axial force in the columns. However, here we practice the determination of AFD from the SFD, starting at joint B shown in the figure below. The direction of the shear force arrows is implied by the SDF shown above.



Equilibrium of joint B in the horizontal direction shows there is no axial force in beam BC. Equilibrium in the vertical direction shows there must be a tensile force equal to V_{BC} in column AB. A similar consideration at C reveals that there must be a compressive force equal to V_{BC} in column CD. In short, the AFD is:



AFD
Tension is positive