

Crack Detection

Here we consider the case of cracks in a weld of a structural member. The cracks have random lengths denoted by the random variable A that has the following probability density function, $f_a = f_A(a)$:

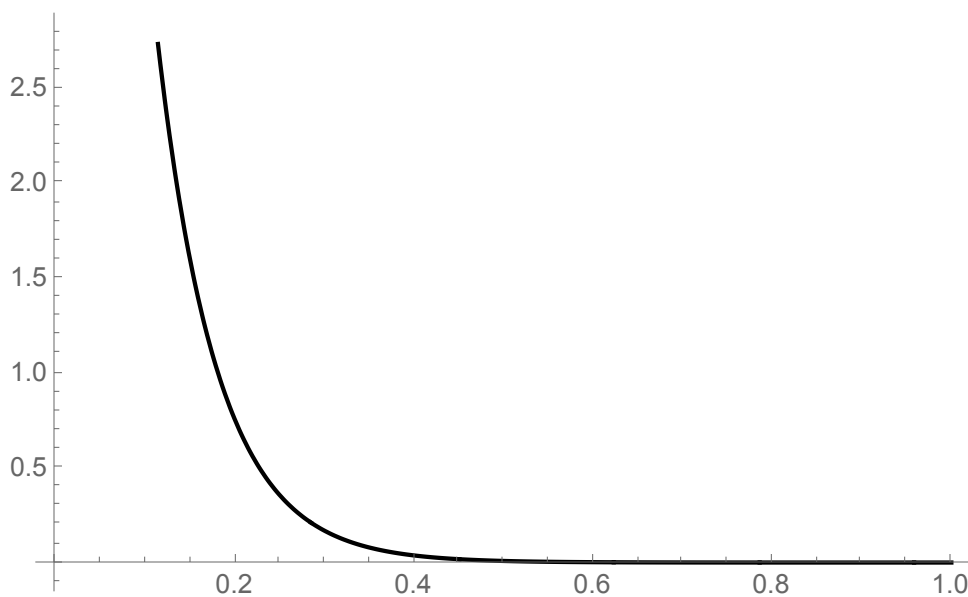
$$f_a = \lambda \text{Exp}[-\lambda a];$$

where the distribution parameter is, in mm^{-1} :

$$\lambda = 15;$$

Here is a plot of that probability distribution:

```
Plot[fa, {a, 0, 1}, PlotStyle -> Black]
```

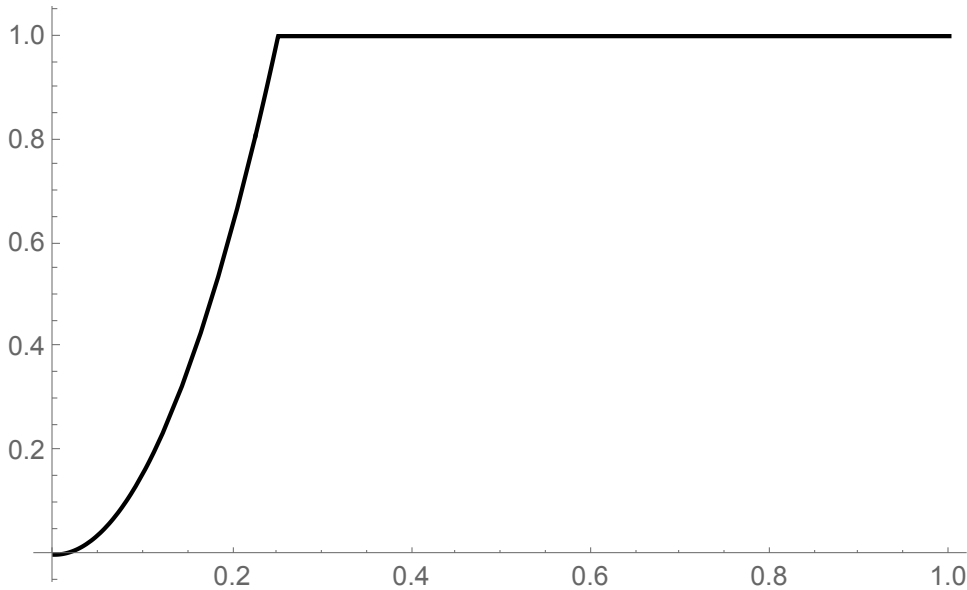


An x-ray device is used to detect the cracks. The probability that a crack will be detected depends on its length and is $PCa = P(C|A=a)$:

$$PCa = \text{If}[a > 0.25, 1, 16 a^2];$$

And here is a plot of that probability as function of a :

```
Plot[PCa, {a, 0, 1}, PlotStyle -> Black]
```



PDF of detected crack

Bayes rule:

$$f_{aC} = \frac{PCa}{PC} f_a;$$

Denominator evaluated by the rule of total probability:

$$PC = \int_0^{\infty} PCa f_a da$$

which yields: 0.126335

That means Bayes rule evaluates to:

$$f_{aC}$$

which yields: $118.732 e^{-15 a}$ If $[a > 0.25, 1, 16 a^2]$

Perhaps that is hard to read, so here it is for $a < 0.25$:

$$f_{aC_{low}} = 118.732 e^{-15 a} 16 a^2$$

which yields: $1899.71 a^2 e^{-15 a}$

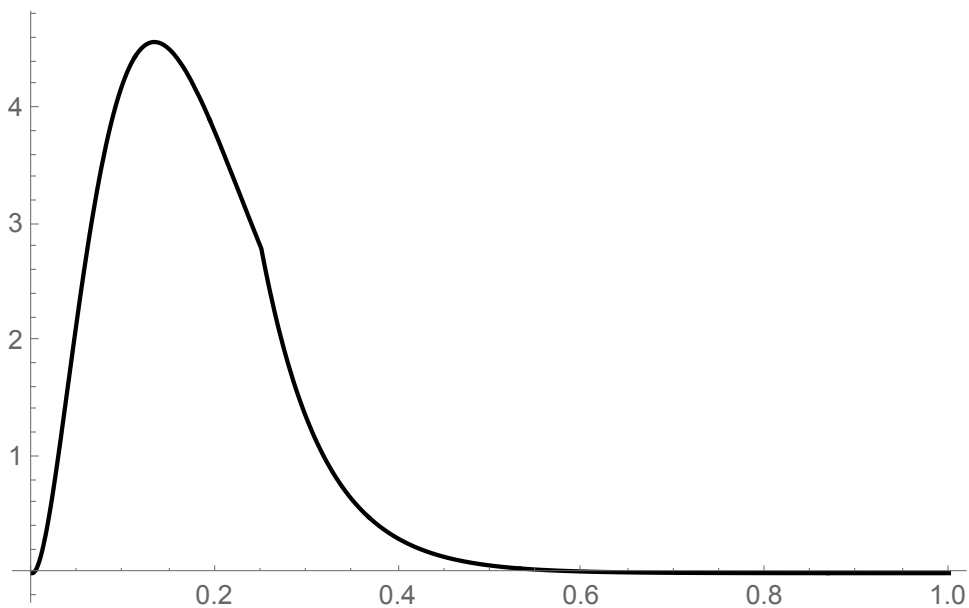
And here it is for $a > 0.25$:

$$fa_{\text{Chigh}} = 118.732 e^{-15 a} 1$$

which yields: $118.732 e^{-15 a}$

And here is a plot of it:

```
Plot[faC, {a, 0, 1}, PlotStyle -> Black]
```



PDF of undetected crack

Bayes rule:

$$fa_{\text{Cbar}} = \frac{1 - PCa}{1 - PC} fa$$

which yields: $17.169 e^{-15 a} (1 - \text{If}[a > 0.25, 1, 16 a^2])$

Perhaps that is hard to read, so here it is for $a < 0.25$:

$$fa_{\text{Cbarlow}} = 17.169 e^{-15 a} (1 - 16 a^2) // \text{Simplify}$$

which yields: $(17.169 - 274.704 a^2) e^{-15 a}$

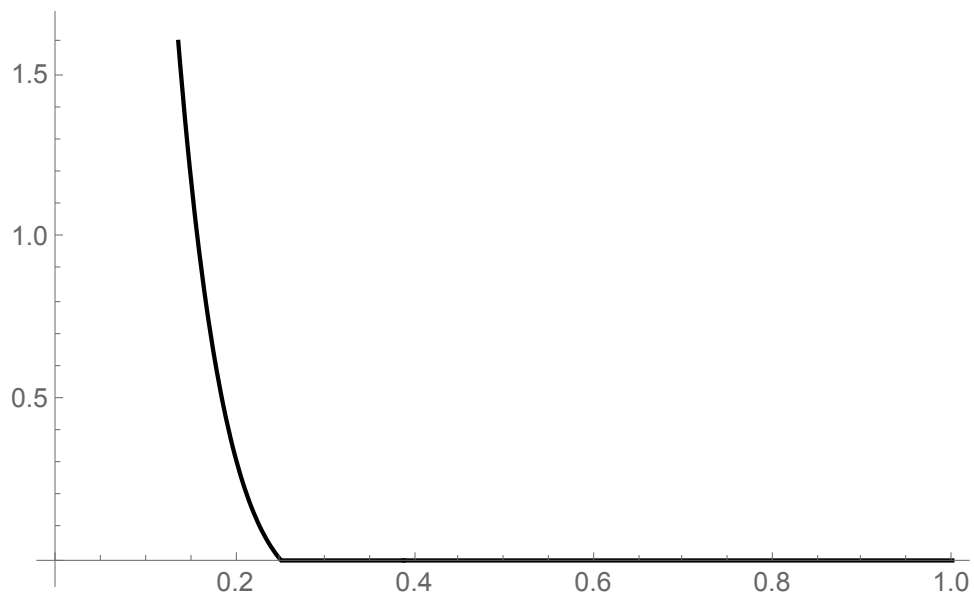
And here it is for $a > 0.25$:

`faCbarhigh = 0`

which yields: 0

And here is a plot of it:

```
Plot[faCbar, {a, 0, 1}, PlotStyle -> Black]
```



Joint plot

```
Plot[{fa, faC, faCbar}, {a, 0, 1}, AxesLabel → {"a", "PDFs"},  
PlotLegends → "Expressions", PlotRange → {{0, 0.4}, {0, 10}}]
```

