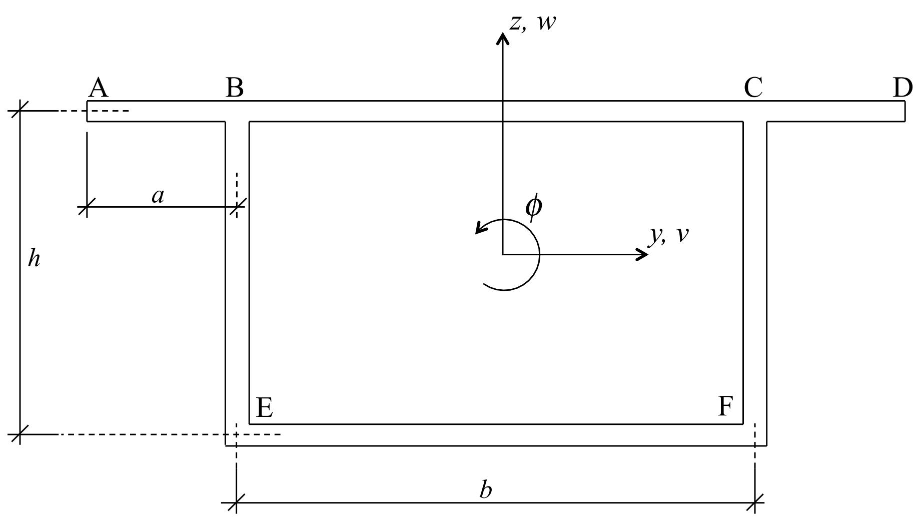
Closed rectangular cross-section with flanges

The cross-section shown in the figure below is considered with the objective of calculating all cross-section constants and determine stresses due to given stress resultants. The cross-section is "thin-walled" and the dimensions *b* and *h* are given to the centre-lines of the cross-section parts. All parts of the cross-section has the thickness *t*.



Input values in N and mm

Dimensions:

```
b = 200;
h = 150;
a = 80;
t = 10;
```

Stress resultants:

```
N = 50;
My = 50;
Mz = 50;
Vz = 50;
Vy = 50;
TStV = 0.5;
B = 0.5;
```

Axial stress due to N

Cross-section area:

$$A = 2\,(a\,t + b\,t + h\,t)$$

which yields: 8600

Stress:

$$\sigma N = \frac{N}{A}\ //\ N$$

which yields: 0.00581395

Axial stress due to My

Location of neutral axis, relative to the mid-line of the bottom part:

$$z0 = \frac{2\,t\,h\,\frac{h}{2} + t\,(b + 2\,a)\,h}{A}\ //\ N$$

which yields: 88.9535

Moment of inertia:

$$IyTopFlange = t\,(b + 2\,a)\,(h - z0)^2;$$
$$IyBottomFlange = t\,b\,z0^2;$$
$$IyWeb = \frac{t\,h^3}{12} + t\,h\left(\frac{h}{2} - z0\right)^2;$$
$$Iy = IyTopFlange + IyBottomFlange + 2\,IyWeb$$

which yields: 3.54506×10^7

Stress:

$$\sigma Mybottom = \frac{My}{Iy}\,z0$$

which yields: 0.000125461

$$\sigma Mytop = \frac{My}{Iy}\,(h - z0)$$

which yields: 0.0000861009

Axial stress due to Mz

Moment of inertia:

$$Iz = \frac{t\,(b + 2\,a)^3}{12} + \frac{t\,b^3}{12} + 2\,t\,h\left(\frac{b}{2}\right)^2 \;//\; N$$

which yields: 7.55467×10^7

Stress:

$$\sigma MzTips = \frac{Mz}{Iz}\left(\frac{b}{2} + a\right)$$

which yields: 0.000119132

$$\sigma MzWebs = \frac{Mz}{Iz}\,\frac{b}{2}$$

which yields: 0.0000661843

Shear stress due to Vz

This is a closed cross-section, hence the shear flow is statically indeterminate, i.e., it cannot be determined by equilibrium alone. Solution: make a cut immediately below C and use a compatibility equation to glue it back together. The unknown shear flow at C is the redundant, named q0, or equivalently the corresponding value of the first moment of area, Q0.

APPROACH 1

One way of computing the redundant is to integrate the first moment of area, Q, of the statically determinate cross-section:

$$Q0 = -\frac{\int \frac{Q}{G\,t}\,ds}{\int \frac{1}{G\,t}\,ds} = -\frac{\int Q\,ds}{\int ds}$$

The statically determinate Q diagram, starting at the free end at D is (note that if we had started at C, the diagram would be the same but with flipped sign):

$$QCdet = t\,a\,(h - z0)$$

which yields: 48 837.2

$$QBdetRight = QCdet + t\, b\, (h - z0)$$

which yields: 170 930.

$$QBdetLeft = t\, a\, (h - z0)$$

which yields: 48 837.2

$$QBdetBelow = QBdetLeft + QBdetRight$$

which yields: 219 767.

$$QNAdetLeft = QBdetBelow + t\, \frac{(h - z0)^2}{2}$$

which yields: 238 401.

$$QEdet = QNAdetLeft - t\, \frac{z0^2}{2}$$

which yields: 198 837.

$$QFdet = QEdet - t\, b\, z0$$

which yields: 20 930.2

$$QNAdetRight = QFdet - 0.5\, t\, z0^2$$

which yields: −18 633.4

Check:

$$QNAdetRight + 0.5\, t\, (h - z0)^2$$

which yields: 4.00178×10^{-11}

Integration (only around the cell) yields:

$$QBE = QBdetBelow + t \int_0^x ((h - z0) - s)\, ds;$$

$$QFC = QFdet - t \int_0^x (z0 - s)\, ds;$$

$$\text{QdetIntegral} = \frac{1}{2}(\text{QCdet} + \text{QBdetRight})\,b + \int_0^h \text{QBE}\,dx + \frac{1}{2}(\text{QEdet} + \text{QFdet})\,b + \int_0^h \text{QFC}\,dx$$

which yields: 7.69186×10^7

$$\text{Q0method1} = -\frac{\text{QdetIntegral}}{2\,b + 2\,h}$$

which yields: $-109\,884.$

Notice that for this single-symmetric cross-section we know that the final Q-value midway between B and C should be zero. In the statically determinate Q-diagram the value is (i.e., this is what the normalizing constant should be):

$$-\frac{1}{2}(\text{QCdet} + \text{QBdetRight})$$

which yields: $-109\,884.$

Same between E and F:

$$-\frac{1}{2}(\text{QEdet} + \text{QFdet})$$

which yields: $-109\,884.$

APPROACH 2

Its value is determined from the integral:

$$\text{q0} = \frac{\text{Vmax}}{\text{Iy}} \int g\,z\,t\,ds$$

In fact, the integral is the reference value of the first moment of area at C:

$$\text{Q0} = \int g\,z\,t\,ds$$

The function g(s) is established by first adding contributions ds/t around the cross-section:

$$\text{gNorm} = 2\,\frac{h}{t} + 2\,\frac{b}{t}$$

which yields: 70

The normalized g-values are then, for different locations around the cross-section:

$$gF = 0 + \frac{h}{t}\frac{1}{gNorm} \;//\; N$$

which yields: 0.214286

$$gE = gF + \frac{b}{t}\frac{1}{gNorm} \;//\; N$$

which yields: 0.5

$$gB = gE + \frac{h}{t}\frac{1}{gNorm} \;//\; N$$

which yields: 0.714286

$$gC = gB + \frac{b}{t}\frac{1}{gNorm}$$

which yields: 1.

To ease the evaluation of the integral for q0, formulas for g are established for each cross-section part:

$$gCF = \frac{gF}{h} s;$$

$$gFE = gF + \frac{gE - gF}{b} s;$$

$$gEB = gE + \frac{gB - gE}{h} s;$$

$$gBC = gB + \frac{gC - gB}{b} s;$$

In fact, it is also useful to establish formulas for z for each cross-section part:

$$zCF = (h - z0) - s;$$

$$zFE = -z0;$$

$$zEB = -z0 + s;$$

$$zBC = (h - z0);$$

The protruding free flanges also contribute to the integral because the shear flow from those free flanges joins the shear flow around the cell. The first moment of area of the free flanges, where they are attached to the cell, is:

```
QfreeFlange = t a (h - z0);
```

Then the full integral is:

$$Q0method2 = \left(\int_0^h gCF\ zCF\ t\ ds + \int_0^b gFE\ zFE\ t\ ds + \int_0^h gEB\ zEB\ t\ ds + gB\ QfreeFlange + \int_0^b gBC\ zBC\ t\ ds + gC\ QfreeFlange\right)$$

which yields: 109 884.

From that reference value, the first moment of area (and thus the shear flow) at other locations around the cell is found by simply adding the redundant (which is constant around the cell) to the Q-values for the statically determinate cross-section:

```
QCrightFinal = QCdet
```

which yields: 48 837.2

```
QCleftFinal = QCdet + Q0method1
```

which yields: -61 046.5

```
QBrightFinal = QBdetRight + Q0method1
```

which yields: 61 046.5

```
QBleftFinal = QBdetLeft
```

which yields: 48 837.2

```
QBbelowFinal = QBdetBelow + Q0method1
```

which yields: 109 884.

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QNAleftFinal = QNAdetLeft + Q0method1
```

which yields: 128 517.

$$\text{QEfinal} = \text{QEdet} + \text{Q0method1}$$

which yields: 88 953.5

$$\text{QFfinal} = \text{QFdet} + \text{Q0method1}$$

which yields: − 88 953.5

$$\text{QNArightFinal} = \text{QNAdetRight} + \text{Q0method1}$$

which yields: − 128 517.

The corresponding shear stress values are:

$$\tau\text{Cright} = \frac{\text{Vz}}{\text{Iy}\ \text{t}}\ \text{QCrightFinal}$$

which yields: 0.00688807

$$\tau\text{Cleft} = \frac{\text{Vz}}{\text{Iy}\ \text{t}}\ \text{QCleftFinal}$$

which yields: − 0.00861009

$$\tau\text{Bright} = \frac{\text{Vz}}{\text{Iy}\ \text{t}}\ \text{QBrightFinal}$$

which yields: 0.00861009

$$\tau\text{Bleft} = \frac{\text{Vz}}{\text{Iy}\ \text{t}}\ \text{QBleftFinal}$$

which yields: 0.00688807

$$\tau\text{Bbelow} = \frac{\text{Vz}}{\text{Iy}\ \text{t}}\ \text{QBbelowFinal}$$

which yields: 0.0154982

$$\tau\text{NAleft} = \frac{\text{Vz}}{\text{Iy}\ \text{t}}\ \text{QNAleftFinal}$$

which yields: 0.0181262

$$\tau E = \frac{Vz}{Iy\,t} QEfinal$$

which yields: 0.0125461

$$\tau F = \frac{Vz}{Iy\,t} QFfinal$$

which yields: −0.0125461

$$\tau NAright = \frac{Vz}{Iy\,t} QNArightFinal$$

which yields: −0.0181262