

Chevalier de Mere's Second Problem

When throwing dice, Chevalier de Mere (Gombaud) expected the same “fifty-fifty” chance of getting one six in four throws with one die (Game 1) as getting two sixes in six times four (24) throws with two dice (Game 2). He argued that in Game 1 there are six possible outcomes in each throw, one of which is favourable. Conversely, in Game 2 there are six times six (36) possible outcomes in each throw, of which one favourable. As a result, intuition suggested that throwing the dice six more times in Game 2 would make the chances equal to Game 1. However, Gombaud had apparently gambled enough to sense that something was wrong...

Game 1

To compute the probability of getting a 6 in 4 rolls with one die, Pascal and Fermat followed the same approach as with Chevalier de Mere's first problem, namely to count possible realizations. When rolling a die four times, the total number of scenarios is

$$\text{numberOfScenariosInGame1} = 6^4$$

which yields: 1296

Intuitively, we may seek the the number of realizations that has the event of interest and divide it by the total number of scenarios. However, while it is not impossible count the number of scenarios that would include a six, it is often easier to follow the complementary approach, namely to count the number of scenarios that would NOT include a six:

$$\text{numberOfScenariosWithoutSix} = 5^4$$

which yields: 625

Then, the probability of getting a six is ONE MINUS the ratio:

$$\text{probabilityInGame1} = 1 - \frac{\text{numberOfScenariosWithoutSix}}{\text{numberOfScenariosInGame1}} // N$$

which yields: 0.517747

Game 2

To compute the probability of getting two 6s in 24 rolls with two dice, the number of possible scenarios is:

$$\text{numberOfScenariosInGame2} = (6 \times 6)^{24}$$

which yields: 22 452 257 707 354 557 240 087 211 123 792 674 816

The number of scenarios that does NOT involve two sixes are:

$$\text{numberOfScenariosWithoutTwoSixes} = ((6 \times 6) - 1)^{24}$$

which yields: 11 419 131 242 070 580 387 175 083 160 400 390 625

Again, the sought probability is ONE MINUS the ratio:

$$\text{probabilityInGame2} = 1 - \frac{\text{numberOfScenariosWithoutTwoSixes}}{\text{numberOfScenariosInGame2}} // N$$

which yields: 0.491404