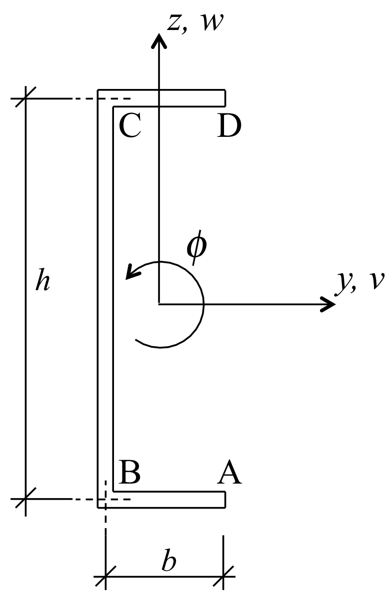
Channel cross-section

The open cross-section shown in the figure below is considered. The objective is to calculate all cross-section constants and determine all stresses due to given stress resultants. The thickness of all parts of the cross-section is denoted *t*. The cross-section is "thin-walled" and notice that the dimensions are given to the centre-lines of the cross-section parts.



Input values

Dimensions:

```
b = 80 mm ;
h = 250 mm ;
t = 10 mm ;
```

Stress resultants:

```
N = 50 kN;
My = 50 m kN;
Mz = 5 m kN;
Vz = 5 kN;
Vy = 5 kN;
TStV = 0.5 m kN;
B = 0.5 kN * m^2;
```

Axial force

The relevant cross-section constant is in this case the cross-sectional area:

```
A = 2 b t + h t
```

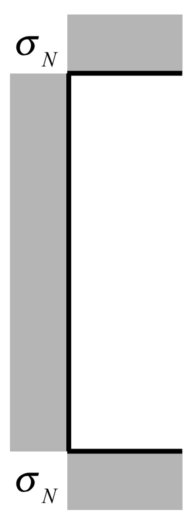
which yields: $4100\ \text{mm}^2$

The axial stress is the axial force divided by the cross-sectional area:

```
σN = N/A;
UnitConvert[σN, "N/mm2"] // N
```

which yields: $12.1951\ \text{N/mm}^2$

The axial stress is distributed uniformly over the cross-section as shown here:



Bending about the y-axis

The relevant cross-section constant is in this case the moment of inertia of the cross-section about the y-axis. Notice that for thin-walled cross-sections it it convenient and sufficiently accurate to neglect the local moment of inertia of cross-section parts that are aligned parallel with the y-axis. For those parts we include only the term dictated by the parallel axis theorem (Steiner's sats), namely area times distance to the neutral axis squared:

$$Iy = \frac{t\,h^3}{12} + 2\,b\,t\left(\frac{h}{2}\right)^2 \;//\; N$$

which yields: $3.80208 \times 10^7\ \text{mm}^4$

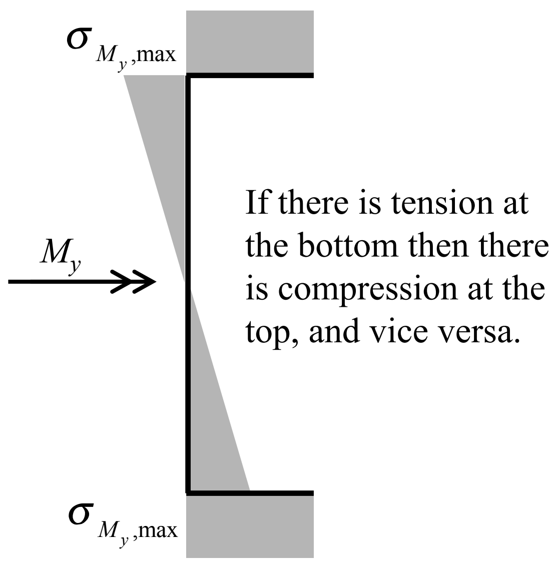
The maximum stress appears at the outermost fibre:

$$\sigma MyMax = \frac{My}{Iy}\frac{h}{2};$$

```
UnitConvert[σMyMax, "N/mm2"]
```

which yields: $164.384\ \text{N/mm}^2$

This stress is distributed over the cross-section as shown here:



Bending about the z-axis

Here we must first determine the location of the neutral axis along the y-axis, relative to the centre-line of the web, denoted y_0 and shown in Figure 4. This distance is the "first moment of area" about that reference point, divided by the total area:

$$y0 = \frac{(2\,b\,t)\,\frac{b}{2}}{A} \;//\; N$$

which yields: 15.6098 mm

The moment of inertia is as follows, again neglecting the local moment of inertia for the cross-section part that is parallel to the z-axis:

$$Iz = \left(\frac{h\,t^3}{12} + h\,t\,y0^2\right) + 2\left(\frac{t\,b^3}{12} + b\,t\left(\frac{b}{2} - y0\right)^2\right)$$

which yields: $2.43514 \times 10^6\ \text{mm}^4$

The axial stress in the web, i.e., at any location from B to C is:

```
σMzB = Mz/Iz y0;
UnitConvert[σMzB, "N/mm2"]
```

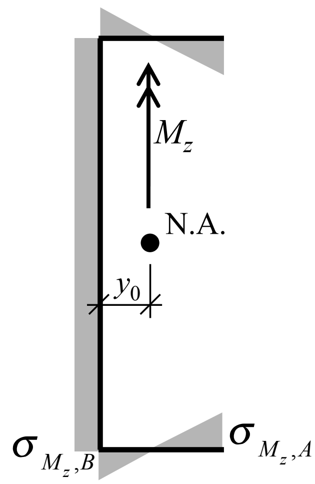
which yields: $32.051\ \text{N/mm}^2$

The axial stress at the tip of each flange is

```
σMzA = Mz/Iz (b - y0);
UnitConvert[σMzA, "N/mm2"]
```

which yields: $132.21\ \text{N/mm}^2$

This axial stress is distributed over the cross-section as shown here:



Shear stress due to shear force in the z-direction

The shear stress is computed by the formula

$$\tau = \frac{V}{I\,t} Q$$

where Q is the first moment of area about the point where the stress is calculated. Using that formula, the shear stress at B is:

$$\tau VzB = \frac{Vz}{Iy\,t}\left(b\,t\,\frac{h}{2}\right);$$

```
UnitConvert[τVzB, "N/mm2"]
```

which yields: $1.31507\ \mathrm{N/mm^2}$

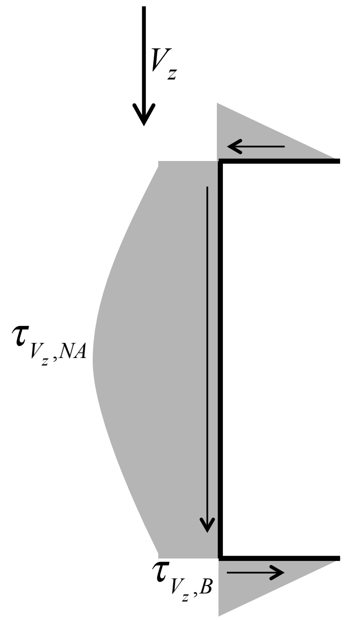
At the neutral axis the shear stress is:

$$\tau VzNA = \tau VzB + \frac{Vz}{Iy\,t}\left(t\,\frac{h}{2}\,\frac{h}{4}\right);$$

```
UnitConvert[τVzNA, "N/mm2"]
```

which yields: $2.34247\ \mathrm{N/mm^2}$

The shear stress is distributed as follows:



Shear centre by equilibrium

Now that we have the shear stress due to a shear force in the z-direction it is possible to determine the shear centre by equilibrium considerations. For that purpose, we first calculate the force resultant from the shear stress in each cross-section part. In the "flanges" that resultant it:

```
VzFlange = 1/2 τVzB t b
```

which yields: 0.526027 kN

In the "web" it is:

```
VzWeb = τVzB t h + 2/3 (τVzNA - τVzB) t h
```

which yields: 5. kN

Then the location of the shear centre, relative to the centre-line of the web, is determined by equilibrium about the shear centre, which is located an unknown distance from the centre of the web:

```
equation = VzFlange h == VzWeb distance;
solution = Solve[equation, distance];
ySC = (distance /. solution)[[1]];
UnitConvert[ySC, "mm"]
```

which yields: 26.3014 mm

Shear stress due to Vy

Using the same approach as above, the shear stress at the neutral axis is

$$\tau\text{VyNA} = \frac{\text{Vy}}{\text{Iz t}}\left(\text{t}\,\frac{(\text{b}-\text{y0})^2}{2}\right);$$

```
UnitConvert[τVyNA, "N/mm2"]
```

which yields: $4.25653\ \text{N}/\text{mm}^2$

Stress at corners:

$$\tau\text{VyB} = \tau\text{VyNA} - \frac{\text{Vy}}{\text{Iz t}}\left(\text{t}\,\frac{\text{y0}^2}{2}\right);$$

```
UnitConvert[τVyB, "N/mm2"]
```

which yields: $4.00638\ \text{N}/\text{mm}^2$

Here we also check that the shear stress at the midpoint of the web is zero:

$$\text{tauVyMidweb} = \text{tauVyCorner} - \frac{\text{Vy}}{\text{Iz t}}\left(\text{t}\,\frac{\text{h}}{2}\,\text{y0}\right)$$

which yields: $\text{tauVyCorner} + -0.00400638\ \text{kN}/\text{mm}^2$

The following figure shows the distribution of the shear stress over the cross-section:

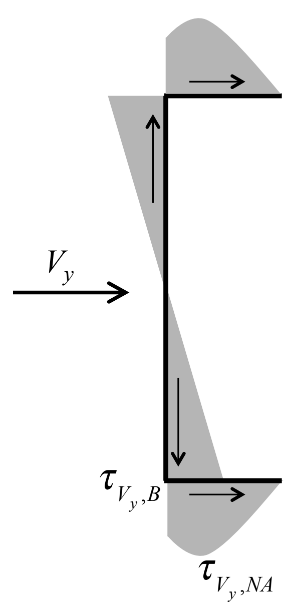


Figure 6: Shear stress due to shear force in the y-direction.

Shear stress due to $T_{St.V}$.

The cross-section constant for Saint Venant torsion is

$$J = \frac{1}{3} h\, t^3 + 2 \times \frac{1}{3} b\, t^3 \; // \; N$$

which yields: 136 667. mm^4

The shear stress is obtained from Prandtl's stress function, which in this case reads

$$\Phi = k \left(1 - 4 \frac{r^2}{t^2}\right);$$

Differentiation with respect to *r* yields the shear stress:

$$\tau StV = \left(\frac{8\, k}{t^2}\right) r;$$

The maximu shear stress appears at the edge:

$$r = \frac{t}{2};$$

The constant *k* is determined from the stress resultant equation T=two times the volume under the stress function:

$$equation = TStV == 2 \times \frac{2}{3} k\, t\, (2\, b + h);$$

Solving yields the following *k*-value:

```
solution = Solve[equation, k];
k = (k /. solution)[[1]];
UnitConvert[k, "N/mm"]
```

which yields: 91.4634 N/mm

In turn, this yields the following maximum shear stress:

```
τStV /. solution;
UnitConvert[τStV, "N/mm2"]
```

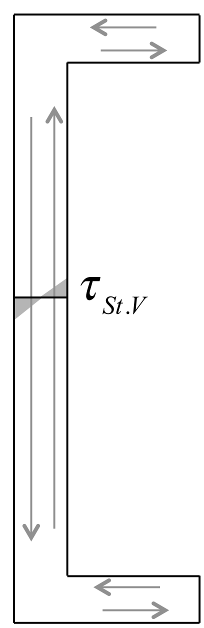
which yields: 36.5854 N/mm^2

Those calculations can be combined into the formula given in the notes, which gives the same shear stress value:

$$\text{UnitConvert}\left[\left(\frac{8\,\frac{t}{2}}{t^2}\right)\left(\frac{3\,\text{TStV}}{4\,t\,(2\,b+h)}\right),\ \text{"N/mm2"}\right]$$

which yields: 36.5854 N/mm^2

The distribution of the shear stress over the cross-section is shown here:



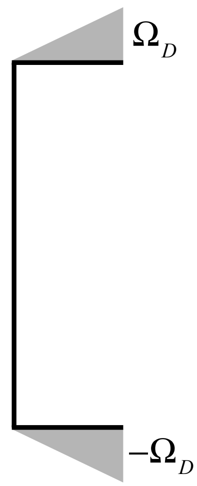
Axial stress due to bi-moment B

A trial omega diagram is first established about the trial point Q, selected at mid-height of the web. The value at D is:

$$\Omega QD = b\,\frac{h}{2}$$

which yields: 10 000 mm^2

The trial omega-diagram is shown here:



The integral of this trial diagram is zero, hence the normalization constant C is zero. However, the integral of $z\ \Omega$ is nonzero:

$$\text{integral} = \left(\frac{h}{2}\right)\left(\frac{1}{2}\, t\, b\, \Omega QD\right) + \left(-\frac{h}{2}\right)\left(-\frac{1}{2}\, t\, b\, \Omega QD\right)$$

which yields: $1\,000\,000\,000\ \text{mm}^5$

That gives the same location of the shear centre coordinate relative to the centre of the web as calculated earlier:

$$\text{yscFromWeb} = -\frac{\text{integral}}{\text{Iy}}$$

which yields: -26.3014 mm

In turn, that gives the following shear centre coordinate relative to the neutral axis:

$$\text{ysc} = -\text{y0} + \text{yscFromWeb}$$

which yields: -41.9111 mm

z_Q and z_{sc} is zero due to symmetry. The final omega diagram is $\Omega = \Omega_Q + (y_{sc} - y_Q)\, z$ where we notice that

```
yQ = -y0;
```

The final omega diagram has the following key values:

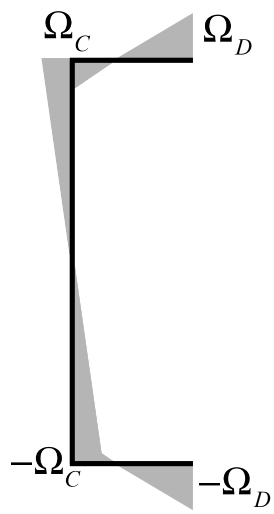
$$\Omega D = \Omega QD + (ysc - yQ)\,\frac{h}{2}$$

which yields: $6712.33\ \text{mm}^2$

$$\Omega C = (ysc - yQ)\,\frac{h}{2}$$

which yields: $-3287.67\ \text{mm}^2$

That omega digram looks like this:



To obtain the cross-sectional constant for warping torsion we use “quick integration formulas” to integrate the omega diagram:

$$Cw = 2\,t\left(\frac{1}{3}\,\Omega C^2\,\frac{h}{2} + \frac{1}{3}\,\Omega C^2\,b + \frac{1}{3}\,\Omega D^2\,b - \frac{2}{6}\,\text{Abs}[\Omega C]\,\text{Abs}[\Omega D]\,b\right)$$

which yields: $2.7032 \times 10^{10}\ \text{mm}^6$

The axial stress is proportional to the omega diagram shown above:

```
σD = B/Cw ΩD;
UnitConvert[σD, "N/mm2"]
```

which yields: $124.155\ \text{N/mm}^2$

```
σC = B/Cw ΩC;
UnitConvert[σC, "N/mm2"]
```

which yields: $-60.8108\ \mathrm{N/mm^2}$