# Concrete Testing

In this document probabilities like P(A) and P(A|B) are written $P_A$ and $P_{A,B}$ to facilitate the calculations. In this problem we suppose we are responsible for testing the concrete mixes delivered to a construction site. Each mix of concrete is delivered by a separate truck, from which samples are taken. Past experience with the concrete manufacturer tells us that the probability that the mix is **G**ood is:

$$P_G = 0.8;$$

Unfortunately, our test method, namely the compression test of concrete cylinders after 7 days of curing, is imperfect. The probability that a **T**est will correctly identify a good quality mix is:

$$P_{T,G} = 0.9;$$

Conversely, the probability that a cylinder-test will erroneously identify a poor mix as being good is:

$$P_{T,\bar{G}} = 0.05;$$

## First test OK

First one cylinder from the mix is tested and it passes the test, i.e., it indicates good concrete. What is the probability that the mix is good? The answer is given by Bayes rule:

$$P_{G,T} = \frac{P_{T,G}}{P_T} P_G;$$

The denominator is calculated by the rule of total probability:

$$P_T = P_{T,G} P_G + P_{T,\bar{G}} (1 - P_G)$$

which yields: 0.73

Result:

$$P_{G,T}$$

which yields: 0.986301

## Second test NOT OK

Now we test an additional cylinder from the same mix but it does NOT pass the test. What is the probability that the mix is good? Using as a prior probability the answer from the previous update:

$$P_G = P_{G,T}$$

which yields: 0.986301

$$P_{G,\overline{T}} = \frac{P_{\overline{T},G}}{P_{\overline{T}}} P_G;$$

$$P_{G,\overline{T}} = \frac{1 - P_{T,G}}{P_{\overline{T}}} P_G;$$

Denominator by the rule of total probability:

$$P_{\overline{T}} = P_{\overline{T},G} P_G + P_{\overline{T},\overline{G}} (1 - P_G);$$

$$P_{\overline{T}} = (1 - P_{T,G}) P_G + \left(1 - P_{T,\overline{G}}\right) (1 - P_G)$$

which yields: 0.111644

Result:

$$P_{G,\overline{T}}$$

which yields: 0.883436