

1D Newton-Raphson Algorithm

The one-parameter version of the algorithm named after Isaac Newton (1643-1727) and Joseph Raphson (1648–1715), implemented in the class *RNewtonLineSearchAlgorithm*, can be derived from a Taylor expansion of the function

$$f(x) = \frac{dF(x)}{dx} = 0 \quad (1)$$

which is the function whose root is sought and $F(x)$ is the objective function in a one-parameter optimization problem

$$x^* = \arg \min \{F(x)\} \quad (2)$$

First-order Taylor approximation of $f(x)$ yields the following version of Eq. (1):

$$f(x) \approx f(x_0) + f'(x_0) \cdot (x - x_0) = 0 \quad (3)$$

Solving for x yields

$$x = x_0 - \frac{f(x_0)}{f'(x_0)} \quad (4)$$

The linearization of $f(x)$ in Eq. (3) implies that x solves $f(x)=0$ when $f(x)$ is linear. When $f(x)$ is nonlinear Eq. (4) is used as a first approximation in a search that repeatedly evaluates Eq. (4) at the previous x -value:

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)} \quad (5)$$