Truss Members

Structural members that take axial force only are called axial bars or truss members. When included in a structural model they have zero bending stiffness. In the following, the theory is established by formulating the equations for equilibrium, material law, and kinematics according the theory of linear elasticity. Because this theory is rather simple, compared with other theories, this document is perhaps the best starting point for understanding the formulation of the structural analysis boundary value problem. In particular, the organization of the theory observed in the section headings carry over to other types of members.

Equilibrium

The equilibrium equation is obtained by considering equilibrium in the *x*-direction for the infinitesimally small length of the truss element shown in Figure 1. This yields:

$$q_x = -\frac{dN}{dx} \tag{1}$$

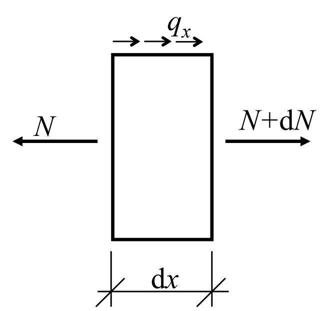

Figure 1: Equilibrium for infinitesimally small truss element.

Section Integration

Because the axial stresses of truss members are uniform over the cross-section, the stress resultant is

$$N = A \cdot \sigma \tag{2}$$

where A is the cross-sectional area.

Material Law

The material law throughout linear elastic theory is Hooke's law:

$$\sigma = E \cdot \varepsilon \tag{3}$$

where E is the modulus of elasticity.

Kinematics

The relationship between the strain and the displacement is obtained by reconsidering the infinitesimal part of the truss member, shown in Figure 2. The dashed line shows the infinitesimal elongation, named *du*.

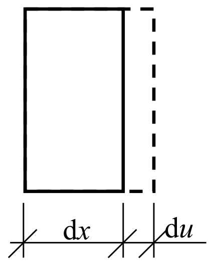

Figure 2: Kinematics for infinitesimally small truss element.

By using the strain definition "elongation divided by original length" the kinematic equation reads:

$$\varepsilon = \frac{du}{dx} \tag{4}$$

Differential Equation

The governing differential equation for truss members is obtained by combining the equations for equilibrium, section integration, material law, and kinematics:

$$q_x = -EA\frac{d^2u}{dx^2} \tag{5}$$

It is also observed that if the equilibrium equations are omitted, then the differential equation reads:

$$N = EA \cdot \frac{\mathrm{d}u}{\mathrm{d}x} \tag{6}$$

General Solution

The differential equation is solved by integrating twice:

$$u(x) = -\frac{q_x}{EA} \cdot x^2 + C_1 \cdot x + C_2 \tag{7}$$

where C_1 and C_2 are integration constants that are determined by the boundary conditions. It is observed that the axial displacement is distributed quadratically along the truss member if a constant traction force, q_x, is applied along the element. In contrast, the displacement is linearly distributed along the element in situations with zero external force along the member.

Cross-section Parameters

The only cross-section constant is the area of the cross-section, A.

Stresses

The stress computation in truss members is trivial because the section integration equation is simple: $\sigma = N/A$.