Fragility Models

Model Description

The terms "fragility curve" or "fragility function" are often interpreted quite broadly, particularly in contemporary performance-based earthquake engineering. Strictly speaking, a fragility curve displays the conditional probability of failure for a component or a system, for given intensity values. Furthermore, a "proper" fragility curves accounts for uncertainty both in capacity and demand. Figure 1 shows a simple visualization of such a fragility curve. Naturally, the probability of failure varies from zero to unity as the intensity increases. Philosophically, a curve like this is the result of a probabilistic analysis, rather than a model that serves as input to the analysis. However, the modern use of the term fragility curve appears to encompass the use of such curves also as models. To understand this concept, consider Figure 2. The figure shows a set of "fragility curves," or more precisely conditional probability curves, that model damage in some building component. The premise is that damage is modelled by a discrete random variable, DS, with outcomes ds_1, ds_2, ds_3, and ds_4. Each curve in Figure 2 displays the probability that the damage is equal to or greater than a specific damage state, for a given demand value. For instance, the left-most curve displays the probability that the damage state is equal to or greater than ds_1. More generally, the probability that the component is in a particular damage state, ds_i, is

$$P(DS = ds_i) = P(DS \geq ds_i) - P(DS \geq ds_{i+1}) \quad (1)$$

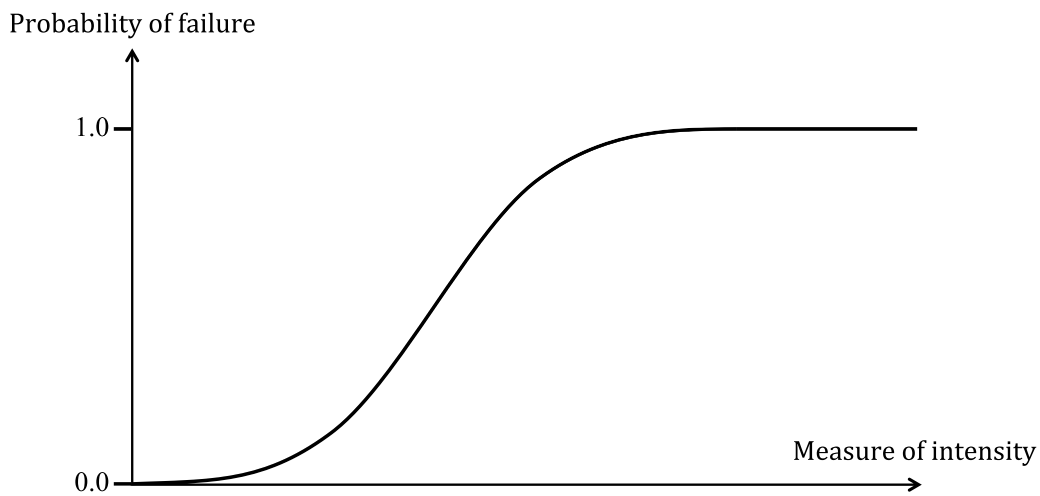

Figure 1: Generic fragility curve.

The modelling of damage with this type of "fragility curves" is appealing from the viewpoint that the curves naturally enter into total probability formulations, such as the "PEER framing equation." It also appears that this type of probabilistic modelling is appreciated by practicing engineers. In fact, today most damage curves like those in Figure 2 are established by expert opinion. It appears natural for experts to express the substantial uncertainty in damage in this way.

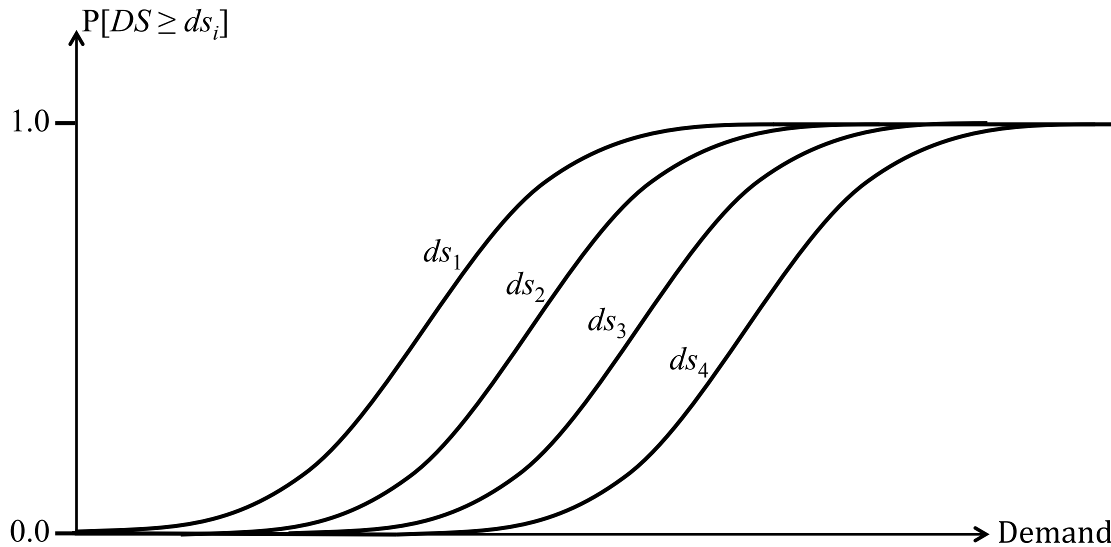

Figure 2: "Fragility functions" for damage of a component.

However, modelling by curves like those in Figure 2 also has disadvantages. First, it limits the probabilistic analysis to total probability integration or sampling. Second, it is in principle necessary to create an entire catalogue of fragility curves to cover different types of components. For example, the damage fragility curves for a four-inch thick wall are presumably different from those of a six-inch thick wall. From this perspective, it appears advantageous to employ regression-type models or more advanced simulation models, which all have a measurable response as output rather than a probability. Not only are such models amenable to reliability analysis, because all uncertainty is modeled by intervening random variables, but each model also covers a range of component types because, say the wall thickness, is an input variable to the model.

Model Parameters

Fragility functions like those in Figure 2 are often described by the lognormal CDF (Applied Technology Council 2009). As shown in the document on continuous random variables, the CDF of a lognormal variable X can be expressed in terms of the standard normal CDF as follows:

$$F(x) = \Phi\left(\frac{1}{\sigma_Y}\ln\left(\frac{x}{m_X}\right)\right) \tag{2}$$

where m_X=median of X and σ_Y=standard deviation of Y=ln(X). With a change in notation to let d=X denote the demand, and applied as a fragility function, Eq. (2) is rewritten:

$$P(DS \geq ds_i) = \Phi\left(\frac{1}{\beta}\ln\left(\frac{d}{\theta}\right)\right) \tag{3}$$

where β=standard deviation of ln(d) and θ=median of d. It is reiterated that ln(d) has the normal distribution and that d thus have the lognormal distribution.

Inference

The fragility function in Eq. (3) has two parameters, β and θ. Suppose N observations of d are available, namely the demand at which the component enters the considered damage state. To determine θ, start by computing the median of the underlying normal random variable, ln(d), which is equal to its mean:

$$m_{\ln(d)} = \mu_{\ln(d)} = \frac{1}{N} \cdot \sum_{i=1}^{N} \ln(d_i) \tag{4}$$

Because $m_{\ln(d)} = \ln(m_d)$, the value of θ, i.e., the median of d, is

$$\theta = m_d = \exp\left(m_{\ln(d)}\right) = \exp\left(\frac{1}{N} \cdot \sum_{i=1}^{N} \ln(d_i)\right) \tag{5}$$

To determine β it is tractable to consider observations of d_i/θ, i.e., a lognormal variable with zero median. As a result, the realizations of $\ln(d_i/\theta)$ are normal with zero mean. In fact, β is indeed the standard deviation of $\ln(d_i/\theta)$. According to classical inference, the variance is:

$$\beta^2 = \text{Var}\left[\ln(d)\right] = \text{Var}\left[\ln\left(\frac{d}{\theta}\right)\right] = \frac{1}{N-1} \cdot \sum_{i=1}^{N} \left(\ln\left(\frac{d_i}{\theta}\right)\right)^2 \tag{6}$$

Thus,

$$\beta = \sqrt{\frac{1}{N-1} \cdot \sum_{i=1}^{N} \left(\ln\left(\frac{d_i}{\theta}\right)\right)^2} \tag{7}$$

Of course, a conceptually simpler approach is to compute the sample mean and sample standard deviation of d_i and thereafter compute the corresponding distribution parameters of whatever distribution is selected, such as the lognormal. Formulas that link the mean and standard deviation to distribution parameters for a variety of distributions are provided in another document on this website.

References

Applied Technology Council. (2009). *Guidelines for Seismic Performance Assessment of Buildings, ATC-58*.