Fractions

In the context of these notes, a fraction represents a division: the numerator is divided by the denominator. The fraction is written with a horizontal or slanted line:

$$\text{Fraction} = \frac{\text{Numerator}}{\text{Denominator}} = \text{Numerator}/\text{Denominator} \tag{1}$$

Numbers that can be written as a fraction with integer numerator and integer denominator are called rational numbers, i.e., if $x, y \in \mathbb{Z}$ then $x/y \in \mathbb{Q}$.

In a proper fraction, the numerator is less than the denominator, in absolute value. The value of a proper fraction is less than 1. A way of expressing an improper fraction is to write it as a mixed number. A mixed number is the sum of an integer (the whole part) and a fraction, written without the plus sign. For $x, y, z \in \mathbb{Z}$:

$$x\tfrac{y}{z} = x + \frac{y}{z} = \frac{x \cdot z + y}{z} \tag{2}$$

An improper fraction is converted to a mixed number by first dividing the numerator by the denominator. The quotient is the whole part and the remainder is the numerator in the mixed number. The denominator stays. Conversely, a mixed number is converted to an improper fraction as shown in Eq. (2).

Two fractions are said to be equivalent when they yield the same value. The multiplication of the numerator and the denominator by the same non-zero number of fraction yields an equivalent fraction. This technique is employed to reduce the fraction. A fraction in which the numerator and denominator have no common factors, except 1, is said to be irreducible. The technique is also employed to create fractions with common denominators so the fractions can be compared, added, or subtracted. After creating the common denominator the numerators are added/subtracted to obtain the resulting fraction. Conversely, an integer multiplied by a fraction multiplies the numerator to obtain the resulting fraction. In a multiplication between two fractions the numerators multiply and the denominators multiply to obtain the resulting fraction. The following abstract examples are intended to explain:

- Repeated pattern right after the decimal point:

$$\begin{aligned} 0.xxxxx\ldots &= \frac{x}{9} \\ 0.xyzxyzxyzxyz &= \frac{xyz}{999} \end{aligned} \tag{3}$$

- Repeated pattern that trails zeros after the decimal point:

$$\begin{aligned} 0.000xxxxxxxx &= \frac{x}{9000} \\ 0.0xyzxyzxyz &= \frac{xyz}{9990} \end{aligned} \tag{4}$$

- Repeated pattern that trails some numbers after the decimal point:

$$0.abcxyzxyzxyzxyz = 0.abc + 0.000xyzxyzxyzxyz = \frac{abc}{1000} + \frac{zyz}{999000} \quad (5)$$

A fraction with a sum in the denominator can possibly be split into partial fractions with a less cumbersome denominator. Various techniques and ad-hoc procedures exist for this purpose.