Expected Utility Theory

Early developments in both probability theory and decision theory were based on applications to gambling. In 1654 Pascal and de Fermat founded probability theory by computing expectations. Then, in 1713 Nicolas Bernoulli formulated the St. Petersburg paradox, in which a game has infinite expected value in theory but not in reality, to demonstrate flaws in the expected cost approach. Daniel Bernoulli solved the paradox in 1738 by introducing the concept of "utility" (Bernoulli 1738). This was later formalized as a decision theory based on several axioms (Von Neumann and Morgenstern 1944). The concept of utility adds a degree of freedom to the expected cost theory because it allows a nonlinear relationship between monetary value and utility. This is a powerful and practically useful concept, because a certain amount of money may have different worth for different people. Thus, the relationship between monetary value and utility, which is referred to as a utility function, is a subjective function. A linear utility function implies that that money and utility are exchangeable and that the decision is based on expected cost. Conversely, nonlinear utility functions represent a means of incorporating the decision maker's subjective risk averseness or propensity for gambling, into the decision-making. Figure 1 shows schematically two utility functions. One is linear (the solid line) and one is nonlinear (the dashed line). It is first observed that the utility function is defined between the monetary extremes; the utility is zero at the worst loss, W, and unity at the best gain, B. The so-called basic reference lottery ticket question is employed to establish the utility function. The following question is asked at different money values, m:

> *At which probability, p, would you prefer to enter into a lottery with probability p of getting B and probability 1-p of getting W, instead of getting this guaranteed amount of money, m?*

The answer, i.e., the probability value p, is the utility value at m. It is noted that if the minimum and maximum utility is not 0 and 1 but more generally α and β then the utility at m is $p\beta+(1-p)\alpha$. As an illustration, consider a decision maker with shallow pockets, i.e., a large loss would be unacceptably damaging. This decision maker is unlikely to enter into the risky lottery unless the probability of getting the best gain B is substantial. This strategy is implied in the dashed utility function in Figure 1. Notice that there is a downside to implementing this risk-averse decision strategy: large resources may be unnecessarily committed to ensuring a safe outcome. Conversely, a decision maker with deep pocket would be able to tackle the unlikely worst loss scenario and adopts the linear utility function in Figure 1. A third option exists: a decision maker who places value on the excitement of a gamble may adopt a nonlinear utility function that is below the straight line in Figure 1. In other words, a gambler will enter into the lottery even when the probability of getting B is relatively low.

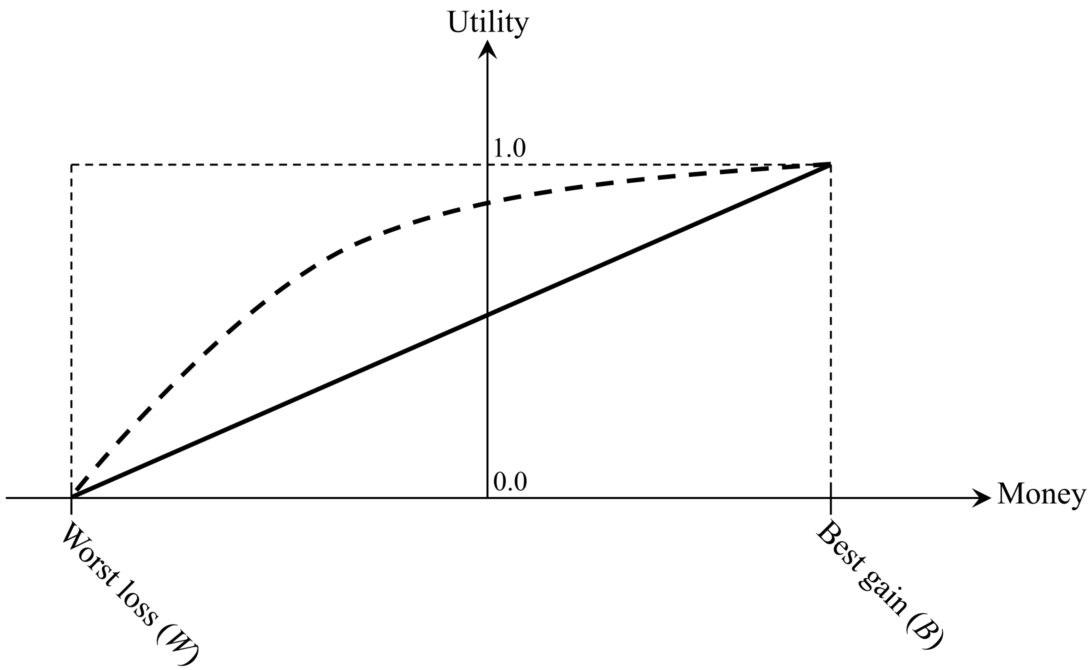

Figure 1: Utility functions.

The presence of multiple competing objectives often seems to complicate and muddle the decision-making process. This may precipitate a deadlock and a sense that the search for one optimal decision is futile. Although methods exist for dealing with multiple objectives, like Pareto techniques, this notion is here rejected and it is here contended that all objectives should be translated into one measure of utility. Once the relationship between money and other costs and benefits are translated into one utility, for example by utility functions, one can employ the techniques of expected cost theory, only with utility instead of cost, to make the decision.

References

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