Expected Cost Theory

A decision made under uncertainty cannot be judged on its outcome. A good decision may seem bad in hindsight due to the occurrence of a low-probability event. That does not mean that it was a bad decision. Conversely, a bad decision may seem good because of luck although it may not have been a good decision. Rather, a good decision is based on a rational and transparent foundation, here referred to as a decision theory. Expected cost theory has the longest history and is still the standard basis for engineering decisions that include cost-benefit considerations.

Discrete Outcome Space

Consider a situation where the outcome of the decision is a discrete variable. One binary example is a gamble where you may either win or not. Another binary example is a structure that may fail or not. In passing it is noted that the presence of discrete outcomes does not influence whether the decision variable is discrete or continuous. The decision variable, such as the dimension of a structural member, can be continuous while the outcome, e.g., failure or not, can be discrete. Problem formulations with a discrete outcome space are common in engineering, and often the outcome is failure or safe. In these situations the total expected cost is

$$\mathrm{E}[c] = c_0(\mathbf{x}) + c_f \cdot p_f(\mathbf{x}) \tag{1}$$

where c_0 is the cost of construction, $\mathbf{x}$ are the decision variables, often called design variables, c_f is the cost of failure, and p_f is the probability of failure. Figure 1 schematically identifies the optimal design for the case of one design variable based on minimization of expected cost. The figure indicates the increasing construction cost due to increasing design variable value, and the decreasing expected cost of failure due to decreasing failure probability as the design variable value increases. The sum of both, which is the total expected cost, is a curve with a unique minimum. This point represents the optimum design and effectively the optimum failure probability. For well-designed structures this curve is relatively flat; small perturbations in the design variable do not lead to significantly increased cost.

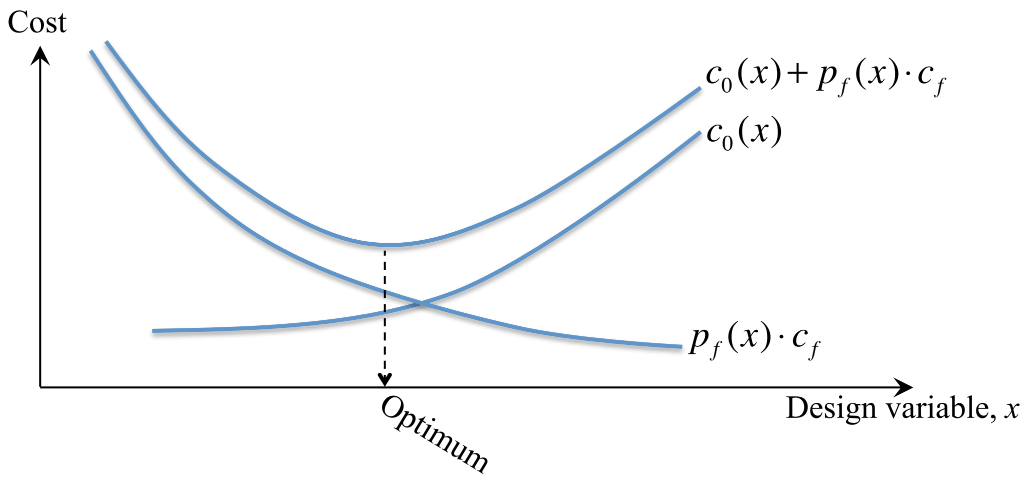

Figure 1: Schematic illustration of reliability-based design optimization.

When constraints on the failure probability and the design variables are included, the reliability-based design optimization problem reads

$$\mathbf{x}^* = \arg\min\left\{c_0(\mathbf{x}) + c_f \cdot p_f(\mathbf{x}) \;\middle|\; \left(p_f(\mathbf{x}) - p_0\right) \leq 0, \quad \mathbf{f}(\mathbf{x}) \leq 0\right\} \tag{2}$$

where the asterisk identifies the optimal design. The first constraint expresses the requirement that the failure probability is less than the threshold p_0. However, one may argue that for reliability-based optimization problems like that in Eq. (2) it is unnecessary to include the probability constraint. This is because the cost of failure is already included in the objective function. If the cost of failure is comprehensively defined, including the potential for human injury and other "costs," then the optimization analysis will provide the optimum target safety. The failure probability will then come out high for a garage, which has a low cost of failure, while it will come out low for a hospital.

Continuous Outcome Space

Consider a problem where the outcome is a continuous random variable. One example is perhaps the total regional cost of damage after an earthquake. Another example is the result of a performance-based earthquake engineering analysis, presented as loss curve. In these situations there are no discrete failure events, which means that the formulation in Eq. (2) is unavailable. However, from the viewpoint of expected cost, the problem is now simpler. The expected value of a random variable is its mean. Thus, the decision basis in this situation is that the optimal design is that which minimizes the mean of the uncertain outcome.