Elasto-plastic Analysis

Elasto-plastic analysis is useful for obtaining estimates of the ultimate capacity of cross-sections and structures. To underpin the calculations it is assumed that all fibres respond to axial stress in accordance with the elasto-plastic material law shown in Figure 1. Although this is a drastic approximation, it facilitates quick "back-of-the-envelope" approximations of the ultimate capacity for many steel and concrete applications, without regard of displacement and damage.

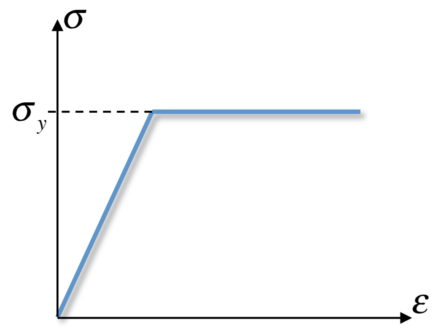

Figure 1: Elasto-plastic material law.

Upper and Lower Bound Theorems

The practical use of elasto-plastic analysis is based on two theorems. The lower bound theorem addresses cross-sections and states that a selected stress distribution will produce a capacity that is less than or equal to the correct value. In other words, the cross-section capacity is either correct or conservative. The upper bound theorem addresses global structures and states that a selected kinematic mechanism will produce a load capacity that is greater than or equal to the true capacity. In other words, when plastic hinges are introduced in a structure to create a "mechanism" the resulting capacity is either correct or un-conservative.

Elasto-plastic Analysis of Cross-sections

The ultimate capacity of a cross-section is determined by assuming the stress distribution. In fact, several different stress distributions can be tried, and the largest is selected as an approximation of the capacity. One example is shown in Figure 2, where a steel I-section is subjected to bending moment and axial force.

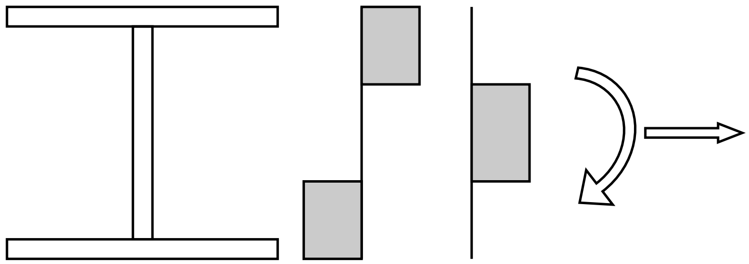

Figure 2: Selected stress distribution in cross-section.

Elasto-plastic Analysis of Frames

The primary challenge when estimating the ultimate capacity of elasto-plastic frames is to determine the correct mechanism, i.e., the appropriate number and location of plastic hinges. The correct choice depends on the specific loading. To obtain a good solution, different mechanisms are tried, and the lowest capacity is taken as a capacity estimate. One plastic hinge configuration is shown in Figure 3 for a portal frame. The capacity associated with this mechanism is obtained by the following procedure:

1. Sketch the deformed shape of the structure for this plastic hinge configuration
2. Determine the internal work carried out by the hinges, which equals the plastic moment, M_p, multiplied by the hinge rotation
3. Determine the external work carried out by the applied load, i.e., force times displacement and moment times rotation
4. Equate the internal and external work and solve for the value of the applied load

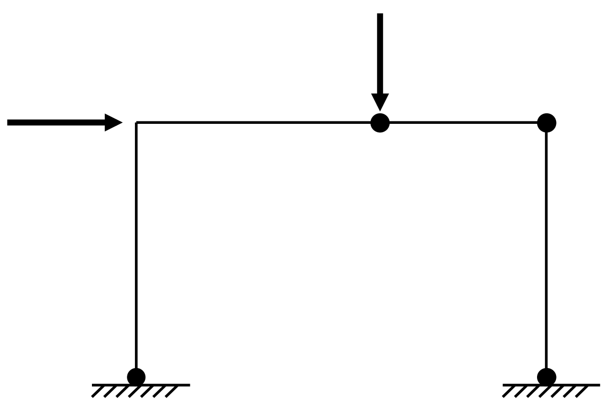

Figure 3: Selected plastic hinges in frame.

Elasto-plastic Analysis of Slabs

The analysis approach for slabs is the same as for frames, but yield lines are assumed instead of plastic hinges. One example is shown in Figure 4, where dashed lines identify the assumed yield lines. Again, different mechanisms are tried and the smallest associated capacity is used as a potentially un-conservative estimate.

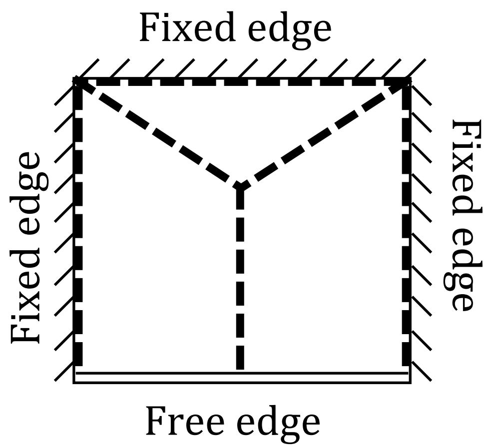

Figure 4: Selected yield lines for slab.