

Bernoulli Sequences

A pedagogical introduction to discrete stochastic processes starts with Bernoulli sequences. These are sequences of trials, e.g., coin tosses, in which each trial yields one of two possible outcomes: success or failure. Importantly, it is assumed that the result of each trial is independent of previous trials, and that the probability of success is constant throughout the trials. Out of n trials, the number of successes, x , is given by the binomial distribution

$$p(x) = \binom{n}{x} \cdot p^x \cdot (1-p)^{n-x} \quad (1)$$

where p is the probability of success in each trial. Another document on mathematics provide details about the evaluation of the binomial coefficient, and the document on univariate distribution types more information about the binomial distribution. That document also shows the geometric distribution, which provides the probability distribution for the number of trials, s , between each success:

$$p(s) = p \cdot (1-p)^{s-1} \quad (2)$$

The mean of s is $1/p$, which is precisely referred to as the mean recurrence time, but often called “return period” for short. Similarly, the negative binomial distribution gives the number of trials until the k^{th} occurrence.

Bayesian Inference

In the Bayesian approach, the model parameter, p , is considered to be a random variable. It turns out that the Beta distribution is the conjugate prior. When x occurrences are observed in n trials then the binomial distribution governs the phenomenon and $p \sim \text{Beta}(a,b)$ is updated according to the rule

$$\begin{aligned} a'' &= a' + x \\ b'' &= b' + n - x \end{aligned} \quad (3)$$

When x trials are observed until the first occurrence then the geometric distribution governs the phenomenon and $p \sim \text{Beta}(a,b)$ is updated according to the rule

$$\begin{aligned} a'' &= a' + 1 \\ b'' &= b' + n - 1 \end{aligned} \quad (4)$$

When x trials are observed until the k^{th} occurrence then the negative binomial distribution governs the phenomenon and $p \sim \text{Beta}(a,b)$ is updated according to the rule

$$\begin{aligned} a'' &= a' + k \\ b'' &= b' + n - k \end{aligned} \quad (5)$$