# Approximate Methods

Modern structural analysis offers a host of methods and computer tools to determine deformations and internal forces in structures. For instance, the finite element method facilitates detailed and accurate analysis of complex structures with a variety of element types. However, although these tools are powerful, they are associated with two significant downside risks: It is easy for structural engineers to lose the "feel" for how structures respond to loads. Furthermore, it is possible that gross errors in the sophisticated computer analysis go undetected. These concerns cannot be overemphasized and contribute to the motivation for understanding approximate analysis.

Another incentive to learn approximate analysis is efficiency. Good structural engineers are able to swiftly establish the integrity of a design, even during meetings with architects and clients. Comprehensive computer analysis is inappropriate under such circumstances. As an illustration of this point is given in the 2010 volume of the EERI Oral History Series, where the famous San Francisco structural engineer Eric Elsesser shares his secret to structural analysis: "You select the points of contraflexure in beams and columns and quickly get to answers that are within 10 percent of a more exact, laborious approach." This method is central in this document, with the phrase "inflection point" employed instead of contraflexure point. An inflection point is where a varying bending moment diagram passes zero, i.e., where the curvature flips from convex to concave or vice versa. Before going into the details, however, an overview of approximate analysis methods is provided. Another example of the importance of approximate analysis is found in the offshore structural engineering sector, where the requirement that "hand calculation groups" check computer calculations was instituted after the collapse of the "Sleipner A" platform during the final construction phase in 1991.

The purpose of approximate analysis is to obtain results quickly that are sufficiently accurate. There exist a variety of methods for this purpose, but the following four approaches deserve particular mention here:

- Guess the location of **inflection points** in beams and columns
- Guess the **end-moment values** in beams and columns
- Apply a **beam analogy** to analyze trusses
- Estimate displaced shapes and apply **energy methods**

The first approaches are described in this document, while other documents describe energy methods, which include the popular Rayleigh-Ritz method.

## Reference Cases

To make good guesses about the location of inflection points and end-moment values it is necessary to relate them to known reference cases. Figure 1 shows six beam cases that are particularly useful; three with distributed load and three with a point-load at mid-span.

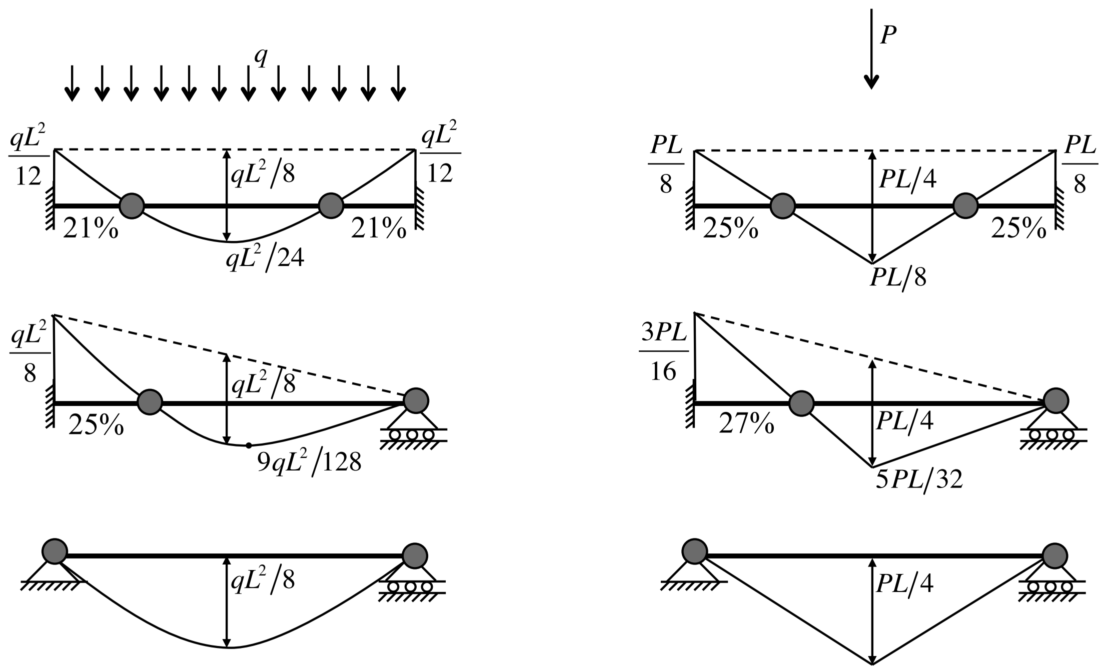


**Figure 1: Reference cases for beams. Circles identify inflection points.**

Consider first the distributed load cases in the left-hand side of Figure 1. Notice that the inflection points for the simply supported beam at the bottom are, naturally, at the ends. Notice further that fixing the left support, as is done in the middle-left beam case in Figure 1, pushes the inflection point out into the span. In the top beam, where both supports are fixed, both inflection points are pushed away from that support. The same tendencies are observed for the left-most beam cases with point load. If the beam were part of a multi-span beam the end-points would neither be fixed nor pinned. In those circumstances, the less bending stiffness is imposed by the neighbouring beam the closer to the support the inflection point will be. These considerations may become clearer by considering bending moments instead. Notice that all the three beam cases to the left in Figure 1 a parabola with height $qL^2/8$ is identified. Differing support conditions simply shifts this parabola up and down. Similarly, for the beam cases at the right-hand side in Figure 1 a triangle with maximum amplitude $PL/4$ is shifted up and down. Consequently, the bending moment in any beam with uniformly distributed load cannot exceed $qL^2/8$.

The guess bending moments or inflection point locations in columns without lateral loads, Figure 2 provides useful reference cases. Notice in particular the left-most case, in which the inflection point is at mid-height. If this were the first-storey column in a multi-storey building then arguably the upper support would not be fixed. Rather, some bending stiffness from adjacent slabs and the column above would provide some stiffness. As a result, the inflection point would move upwards. Not as far up as the right-most case in Figure 2, which is free at the top, but nonetheless, it would move up. Usually, it is assumed that it moves from 50% up to about 60% of the column height. The bottom case in Figure 2 is useful in several beam and frame problems.

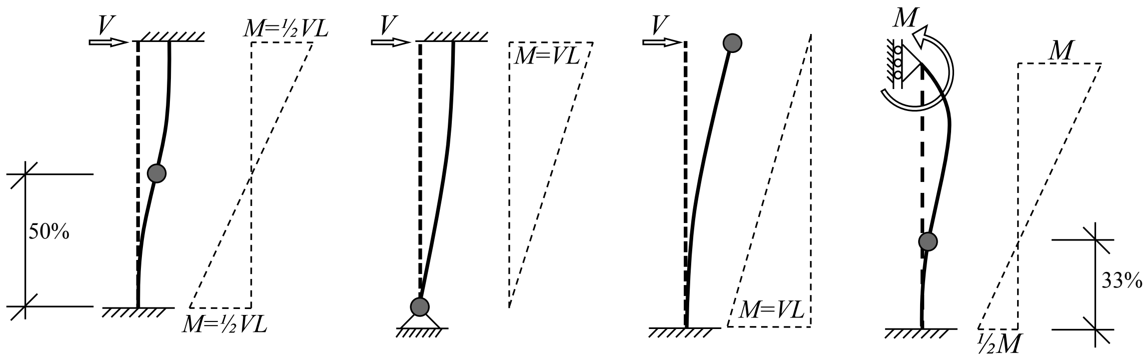


**Figure 2: Reference cases for columns. Circles identify inflection points.**

# Guessing Inflection Points

The purpose with guessing the location of inflection points is that hinges are placed at those locations. With sufficiently many hinges the structure is made statically determinate and subject to simple statics rather than cumbersome methods for indeterminate structures. In other words, the significance of an inflection point is that the bending moment is zero there, thus a hinge at that location does not alter the structural response. Typically, but not always, the number of hinges that are introduced equals the structure's degree of static indeterminacy. Details are provided below for mid-rise and high-rise frame buildings. Continuous beams and simple portal frames are also readily analyzed by guessing inflection points, but further details are not provided here. The technique is essentially to identify similarities between the specific structure and the reference cases in Figure 1 and Figure 2. Proficiency is developed by practice.

### Portal Method for Low-rise Multi-storey Frames

This method is developed for mid-rise multi-storey frames subjected to lateral load. The key assumption is that the lateral load is distributed as shear force onto the columns according to their relative stiffness. Usually it is assumed that the shear forces in interior columns are twice as large as those of exterior columns because interior columns have double moment of inertia compared to the outer columns. This assumption is readily modified. Furthermore, according to Figure 2 it is assumed that a point of inflection is located at the mid-height of each column. This may be modified for the top and bottom columns. It is also assumed that a point of inflection is located at the mid-span of each girder. Based on these assumptions, the following procedure is formulated:

1. Start at the top of the frame and work downwards
2. Pass a horizontal imaginary section between two floors at the height of the inflection points; only shear and axial forces act on these column cross-sections
3. The total lateral shear force above the imaginary section is distributed onto the columns according to relative stiffness, usually twice as much on interior columns
4. Compute bending moments at column ends
5. Compute bending moments at beam-ends by equilibrium at the joints. Start at an exterior joint. Because inflection points are assumed at mid-span the end moments of each beam are equal.

6. Compute shear forces in beams by dividing the sum of the beam end moments by the span lengths
7. The axial force in a column equals the sum of shear forces in all beams above it

### Cantilever Method for High-rise Multi-storey Frames

This method is developed for high-rise multi-storey frames subjected to lateral loads. As in the portal method, inflection points are assumed at mid-span of columns and beams. However, in this method the global structure is considered as one vertical cantilevered beam. The cross-section of this cantilevered beam is composed of the cross-section of the columns. The moment of inertia of the global cantilevered beam is

$$I = \sum_{i=1}^{K} z_i^2 A_i \tag{1}$$

where $K$ is the number of columns in each storey, $z_i$ is the distance from the neutral axial of the global cross-section to column number $i$, and $A_i$ is the cross-sectional area of column number $i$. The beam analogy also provides the axial force, $N_i$, in each column due to the global bending moment, $M$, due to the lateral loads. For this purpose, consider the stress equation from Euler-Bernoulli beam theory:

$$\sigma_i = \frac{M}{I} z_i \tag{2}$$

where $\sigma_i$ is the stress in column number $i$, which gives the following axial force:

$$N_i = \sigma_i A_i \tag{3}$$

In summary, the following procedure is suggested in the cantilever method:

1. Start at the top of the frame and work downwards
2. Pass a horizontal imaginary section between any two floors at the height of the inflection points; only shear and axial forces act on these column cross-sections
3. Compute the global overturning moment, $M$, from the lateral loads; tension on the right-hand side is positive
4. Compute the axial force, $N_i$, in each column according to Eqs. (1), (2), and (3)
5. The shear force in each beam equals the difference in axial load between columns
6. The end-moments in the beams is computed from the shear force: V=2M/L→M=VL/2
7. The bending moment in the columns are obtained from equilibrium at the joints

# Span Lengths

In addition to the approximate methods described above, it is often useful to apply experience to judge of a design is suitable. One of several rules-of-thumb is to review the "span-to-depth ratio," i.e., the span length, $L$, over the cross-section height (depth), $d$. For different structural systems, the following tables suggest typical values, obtained by considering a variety of typical situations, considering both ultimate and serviceability limit-states.

**Table 1: Timber**

| | *L* [m] | *L/d* |
|---|---|---|
| Lumber joist floors | 2-7 | 15-25 |
| I-joist floors | 5-10 | 15-25 |
| Timber beams | 3-10 | 10-20 |
| Glulam beams | 6-25 | 15-20 |
| CLT floors | ? | 20-30 |
| CLT roofs | ? | 30-40 |
| Roof trusses, pitched | 10-40 | 2-8 |
| Roof trusses, flat | 10-40 | 8-15 |

**Table 2: Steel**

| | *L* [m] | *L/d* |
|---|---|---|
| Wide-flange beams | 3-20 | 15-25 |
| Plate (high web) girders | 10-25 | 15-20 |
| Trusses | 10-50 | 8-16 |

**Table 3: Reinforced concrete**

| | *L* [m] | *L/d* |
|---|---|---|
| Slabs | 3-8 | 30-35 |
| T-beams | 3-15 | 8-14 |

**Table 4: Pre-stressed concrete**

| | *L* [m] | *L/d* |
|---|---|---|
| Slabs | 6-10 | 40-44 |
| Beams | 8-20 | 16-28 |
| Double tees (Ts) | 8-30 | 24-36 |